02

The value of a physical quantity is generally expressed as the product of a number and its unit. The unit is simply a particular example of the quantity concerned which is used as a reference.
e.g. The speed $v$ of a particle may be expressed as $v = 25$ m / s = 90 km/h, where metre per second and kilometre per hour are alternative units for expressing the same value of the

# UNITS AND MEASUREMENTS

**CHAPTER CHECKLIST**

- Physical Quantities
- Physical Unit
- Significant Figures and Rounding off
- Dimensional Formulae and Dimensional Equations

## |TOPIC 1|

## System of Units

In order to define units for all fundamental quantities or base quantities, we use the term fundamental units or base units and then to define units for all other quantities as products of powers of the base units that we call derived units. Hence, a complete set of these units, i.e. both the base units and derived units, is known as the system of units.

## PHYSICAL QUANTITIES

All those quantities which can be measured directly or indirectly and in terms of which the laws of physics can be expressed are called physical quantities.

Physical quantities can be further divided into two types:

### (i) Fundamental Quantities

Those physical quantities which are independent of other physical quantities and are not defined in terms of other physical quantities, are called fundamental quantities or base quantities.

e.g. Mass, length, time, temperature, luminous intensity, electric current and the amount of substance, etc.

### (ii) Derived Quantities

Those quantities which can be derived from the fundamental quantities are called derived quantities. e.g. Velocity, acceleration and linear momentum, etc.

# MEASUREMENT OF PHYSICAL QUANTITIES

The measurement of physical quantity is the process of comparing this quantity with a standard amount of the physical quantity of the same kind, called its unit.

Hence, to express the measurement of a physical quantity, we need to know two things

(i) The unit in which the quantity is measured.

(ii) The numerical value or the magnitude of the quantity($n$), i.e. the number of times that unit ($u$) is contained in the given physical quantity.

or $$\boxed{Q = nu}$$

**Numerical Value Inversely Proportional to the Size of Unit**

The numerical value ($n$) is inversely proportional to the size ($u$) of the unit.

$$n \propto \frac{1}{u} \Rightarrow nu = \text{constant}.$$

e.g. The magnitude of a quantity remains the same, whatever may be the unit of measurement.

Hence, 1 kg = 1000 g

We may write as $Q = n_1 u_1 = n_2 u_2$

where $n_1, n_2$ are the numerical values and $u_1, u_2$ are the two units of measurement of the same quantity.

# PHYSICAL UNIT

The standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a physical unit.

The essential requirements of physical unit are given as below:

(i) It should be of suitable size.

(ii) It should be easily accessible.

(iii) It should not vary with time.

(iv) It should be easily reproducible.

(v) It should not depend on physical conditions like pressure, volume, temperature, etc.

The physical unit can be classified into two ways that can be given as below :

## Fundamental Units

The physical units which can neither be derived from one another, nor they can be further resolved into more simpler units are called fundamental units. The units of fundamental quantities, i.e. length, mass and time are called fundamental units or base units.

## Derived Units

The units of measurement of all other physical quantities which can be obtained from fundamental units are called derived units.

e.g. $$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\therefore \quad \text{Unit of speed} = \frac{\text{Unit of distance}}{\text{Unit of time}}$$

$$= \frac{\text{m}}{\text{s}} = \text{ms}^{-1}$$

Unit of speed (i.e. $\text{ms}^{-1}$) is a derived unit.

# THE INTERNATIONAL SYSTEM OF UNITS

A system of units is the complete set of units, both fundamental and derived for all kinds of physical quantities.

The common systems of units used in mechanics are given below

(i) **FPS System** It is the British engineering system of units, which uses foot as the unit of length, pound as the unit of mass and second as the unit of time.

(ii) **CGS System** It is based on Gaussian system of units, which uses centimetre, gram and second for length, mass and time, respectively.

(iii) **MKS System** It uses metre, kilogram and second as the fundamental units of length, mass and time, respectively.

(iv) **International System of Units** (SI Units) The system of units, which is accepted internationally for measurement is the System Internationaled' Units (French for International System of Units) abbreviated as SI.

The SI, with standard scheme of symbols, units and abbreviations, was developed and recommended by **General Conference on Weights and Measures** in 1971 for international usage in scientific, technical, industrial and commercial work.

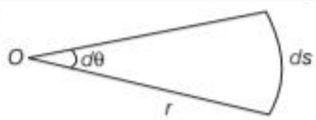
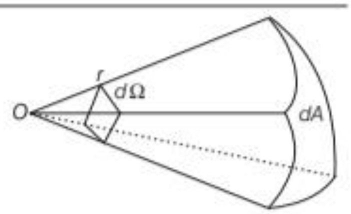
This system of units makes revolutionary changes in the MKS system and is known as **rationalised MKS system**. It is helpful to obtain all the physical quantities in Physics.

In SI system, there are seven base units and two supplementary units as listed below:

**SI base quantities and units**

| Base quantity | SI units | | |
|---|---|---|---|
| | **Name** | **Symbol** | **Definition** |
| Length | Metre | m | One metre is the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second. (1983) |
| Mass | Kilogram | kg | One kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at international Bureau of Weights and Measures at Sevres, near Paris, France. (1889) |
| Time | Second | s | One second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. (1967) |
| Electric current | Ampere | A | One ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce a force between these conductors equal to $2 \times 10^{-7}$ N/m of length. (1948) |
| Thermodynamic temperature | Kelvin | K | One kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967) |
| Amount of substance | Mole | mol | One mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12. (1971) |
| Luminous intensity | Candela | cd | One candela is the luminous intensity in a given direction of a source that emits monochromatic radiation of frequency $540 \times 10^{12}$ Hz and that has a radiant intensity in that direction of 1/683 watt per steradian. (1979) |

**Some supplementary quantities and their SI units**

| Supplementary quantity | SI units | | |
|---|---|---|---|
| | **Name** | **Symbol** | **Definition** |
| Plane angle | Radian | rad | One radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle. i.e. $d\theta = \frac{ds}{r}$   |
| Solid angle | Steradian | sr | One steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere, which is equal in area, to the square of radius of the sphere. i.e. $d\Omega = \frac{dA}{r^2}$   |

**Note**

- The FPS system is not a metric system. This system is not in much use these days.
- The drawback of CGS system is that many of the derived units on this system are inconveniently small.
- The advantages of MKS system is that some of the derived units are of convenient size.

**SI derived units with special names**

| Physical Quantity | SI unit | | | |
|---|---|---|---|---|
| | Name | Symbol | Expression in terms of other units | Expression in terms of SI base units |
| Frequency | Hertz | Hz | – | $s^{-1}$ |
| Force | Newton | N | – | $kg\ m\ s^{-2}$ or $kg\ m/s^2$ |
| Pressure, stress | Pascal | Pa | $N/m^2$ or $N\ m^{-2}$ | $kg\ m^{-1}s^{-2}$ or $kg/s^2\ m$ |
| Energy, work, quantity of heat | Joule | J | Nm | $kg\ m^2s^{-2}$ or $kg\ m^2/s^2$ |
| Power, radiant flux | Watt | W | J/s or $J\ s^{-1}$ | $kg\ m^2\ s^{-3}$ or $kg\ m^2/s^3$ |
| Quantity of electricity, electric charge | Coulomb | C | – | As |
| Electric potential, potential difference, electromotive force | Volt | V | W/A or $WA^{-1}$ | $kg\ m^2\ s^{-3}\ A^{-1}$ or $kg\ m^2/s^3\ A$ |
| Capacitance | Farad | F | C/V | $A^2\ s^4\ kg^{-1}m^{-2}$ |
| Electric resistance | Ohm | Ω | V/A | $kg\ m^2\ s^{-3}A^{-2}$ |
| Conductance | Siemens | S | A/V | $m^{-2}\ kg^{-1}s^3\ A^2$ |
| Magnetic flux | Weber | Wb | Vs or J/A | $kg\ m^2s^{-2}\ A^{-1}$ |
| Magnetic field, magnetic flux density, magnetic induction | Tesla | T | $Wb\ /\ m^2$ | $kg\ s^{-2}\ A^{-1}$ |
| Inductance | Henry | H | Wb/A | $kg\ m^2s^{-2}A^{-2}$ |
| Luminous flux, luminous power | Lumen | lm | – | cd/sr |
| Illuminance | Lux | lx | $lm/m^2$ | $m^{-2}\ cd\ sr^{-1}$ |
| Activity (of a radio nuclide/ radioactive source) | Becquerel | Bq | – | $s^{-1}$ |
| Absorbed dose, absorbed dose index | Gray | Gy | J/kg | $m^2/s^2$ or $m^2\ s^{-2}$ |

**Some SI derived units expressed by means of SI units**

| Physical quantity | SI unit | | |
|---|---|---|---|
| | Name | Symbol | Expression in terms of SI base units |
| Magnetic moment | Joule per tesla | $JT^{-1}$ | $m^2$ A |
| Dipole moment | Coulomb metre | C m | s A m |
| Dynamic viscosity | Poiseuille or pascal second or newton second per square metre | Pl or Pa s or N s $m^{-2}$ | $m^{-1}$ kg $s^{-1}$ |
| Torque, couple, moment of force | Newton metre | N m | $m^2$ kg $s^{-2}$ |
| Surface tension | Newton per metre | N/m | kg $s^{-2}$ |
| Power density, irradiance, heat flux density | Watt per square metre | W/$m^2$ | kg $s^{-3}$ |
| Heat capacity, entropy | Joule per kelvin | J/K | $m^2$ kg $s^{-2}$ $K^{-1}$ |
| Specific heat capacity, specific entropy | Joule per kilogram kelvin | J/kg K | $m^2$ $s^{-2}$ $K^{-1}$ |
| Specific energy, latent heat | Joule per kilogram | J/kg | $m^2$ $s^{-2}$ |
| Radiant intensity | Watt per steradian | W $Sr^{-1}$ | kg $m^2$ $s^{-3}$ $Sr^{-1}$ |
| Thermal conductivity | Watt per metre kelvin | W $m^{-1}$ $K^{-1}$ | m kg $s^{-3}$ $K^{-1}$ |
| Energy density | Joule per cubic metre | J/$m^3$ | kg $m^{-1}$ $s^{-2}$ |
| Electric field strength | Volt per metre | V/m | m kg $s^{-3}$ $A^{-1}$ |
| Electric charge density | Coulomb per cubic metre | C/$m^3$ | $m^{-3}$ A s |
| Electricity flux density | Coulomb per square metre | C/$m^2$ | $m^{-2}$ A s |
| Permittivity | Farad per metre | F/m | $m^{-3}$ $kg^{-1}$ $s^4$ $A^2$ |
| Permeability | Henry per metre | H/m | m kg $s^{-2}$ $A^{-2}$ |
| Molar energy | Joule per mole | J/mol | $m^2$ kg $s^{-2}$ $mol^{-1}$ |
| Angular momentum, Planck constant | Joule second | J s | kg $m^2$ $s^{-1}$ |
| Molar entropy, molar heat capacity | Joule per mole kelvin | J/mol K | $m^2$ kg $s^{-2}$ $K^{-1}$ $mol^{-1}$ |
| Exposure (X-rays and $\gamma$-rays) | Coulomb per kilogram | C/kg | $kg^{-1}$ s A |
| Absorbed dose rate | Gray per second | Gy/s | $m^2$ $s^{-3}$ |
| Compressibility | Per pascal | $Pa^{-1}$ | m $kg^{-1}$ $s^2$ |
| Elastic moduli | Newton per square metre | N/$m^2$ or N $m^{-2}$ | kg $m^{-1}$ $s^{-2}$ |
| Pressure gradient | Pascal per metre | Pa/m or N $m^{-3}$ | kg $m^{-2}$ $s^{-2}$ |
| Surface potential | Joule per kilogram | J/kg or N m/kg | $m^2$ $s^{-2}$ |
| Pressure energy | Pascal cubic metre | Pa $m^3$ or N m | kg $m^2$ $s^{-2}$ |
| Impulse | Newton second | N s | kg ms $s^{-1}$ |
| Angular impulse | Newton metre second | N m s | kg $m^2$ $s^{-1}$ |
| Specific resistance | Ohm metre | Ω m | kg $m^3$ $s^{-3}$ $A^{-2}$ |
| Surface energy | Joule per square metre | J/ $m^2$ or N/m | kg $s^{-2}$ |

# PROBLEM SOLVING STRATEGY (UNIT CONVERSION)

(i) **Identify the relevant concept** Unit conversion is important to recognise what it's needed. In most cases, you are best off using the fundamental SI units (length in metres, mass in kilograms and time in seconds) within a problem.

(ii) **Set up the problem** Units are multiplied and divided just like ordinary algebraic symbols. It gives us an easy way to convert a quantity from one set of units to another.

(iii) **Execute the problem** e.g., We say that 1 min = 60 s. So, the ratio of (1 min)/(60 s) equal to 1/60, as does its reciprocal (60 s)/(1 min). We may multiply a quantity by either of these factors without changing the quantity's by physical meaning. To find the number of seconds in 3 min, we write as

$$3\text{ min} = (3\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right) = 180\text{ s}$$

(iv) **Evaluate your answer** Check whether your answer is reasonable. Is the result 3 min = 180 s reasonable? Is your answer is consistent with an unit of conversion?

**EXAMPLE |1| Conversion of Units**

Calculate the angle of

(i) 1° (degree)

(ii) 1′′ (second of arc or arc sec) in radian. **[NCERT]**

***Sol.*** (i) $1° = \frac{2\pi}{360}\text{ rad} = \frac{\pi}{180} = \frac{22}{7\times 180}$

$= 1.746\times 10^{-2}\text{ rad}$

(ii) $1\text{ arc sec} = 1'' = \frac{1'}{60} = \frac{1°}{60\times 60}$

$= \frac{1}{60\times 60}\times\frac{\pi}{180}\text{ rad}$

$= 4.85\times 10^{-6}\text{ rad}$

***Rules for Writing SI Units***

- Small letters are used for symbols of units.
- Symbols are not followed by a full stop.
- The initial letter of a symbol is capital only when the unit is named after a scientist.
- The full name of a unit always begins with a small letter even if it has been named after a scientist.
- Symbols do not take plural form.

# ADVANTAGES OF SI OVER OTHER SYSTEMS OF UNITS

(i) **SI is a coherent system of units** All derived units can be obtained by simple multiplication or division of fundamental units without introducing any numerical factor.

(ii) **SI is a rational system of units** It uses only one unit for a given physical quantity. e.g, all forms of energy are measured in joule. On the other hand, in MKS system, the mechanical energy is measured in joule, heat energy in calorie and electrical energy in watt hour.

(iii) **SI is a metric system** The multiples and submultiples of SI units can be expressed as powers of 10.

i.e. $a\times 10^{\pm b}$.

(iv) **SI is an absolute system of units** It does not use gravitational units. The use of '*g*' is not required.

**Some General Units (Outside from SI)**

| Name | Symbol | Value in SI Unit |
|---|---|---|
| Minute | min | 60 s |
| Hour | h | 60 min = 3600 s |
| Day | d | 24 h = 86400 s |
| Year | y | 365.25 d = $3.156\times 10^7$ s |
| Degree | ° | 1° = (π/180) rad |
| Litre | L | 1 $dm^3 = 10^{-3} m^3$ |
| Tonne | t | $10^3$ kg |
| Carat | c | 200 mg |
| Bar | bar | 0.1 MPa = $10^5$ Pa |
| Curie | Ci | $3.7\times 10^{10} s^{-1}$ |
| Roentgen | R | $2.58\times 10^{-4}$ C/kg |
| Quintal | q | 100 kg |
| Barn | b | 100 $fm^2 = 10^{-28} m^2$ |
| Area | a | 1 $dam^2 = 10^2 m^2$ |
| Hectare | ha | 1 $hm^2 = 10^4 m^2$ |
| Standard atmospheric pressure | atm | 101325 Pa = $1.013\times 10^5$ Pa |

# ABBREVIATIONS IN POWERS OF TEN

When the magnitudes of the physical quantities are very large or very small, it is convenient to express them in the multiples or submultiples of the SI units.

The various prefixes used for powers of 10 are listed below in table

**Prefixes for Powers of Ten**

| Multiple | Prefix | Symbol | Sub-multiple | Prefix | Symbol |
|---|---|---|---|---|---|
| $10^1$ | deca | da | $10^{-1}$ | deci | d |
| $10^2$ | hecto | h | $10^{-2}$ | centi | c |
| $10^3$ | kilo | k | $10^{-3}$ | milli | m |
| $10^6$ | mega | M | $10^{-6}$ | micro | μ |
| $10^9$ | giga | G | $10^{-9}$ | nano | n |
| $10^{12}$ | tera | T | $10^{-12}$ | pico | p |
| $10^{15}$ | peta | P | $10^{-15}$ | femto | f |
| $10^{18}$ | exa | E | $10^{-18}$ | atto | a |

e.g. 1 megaohm, $M\Omega = 10^6\ \Omega$
1 milliampere or 1 mA $= 10^{-3}$ A
1 kilometre, 1 km $= 10^3$ m
1 microvolt or $1\mu V = 10^{-6}$ V
1 decagram, 1 da g $= 10$ g
1 nanosecond or 1 ns $= 10^{-9}$ s
1 centimetre, 1 cm $= 10^{-2}$ m
1 picofarad or 1 pF $= 10^{-12}$ F

> ***CGS, MKS and SI are Decimal Systems of Units***
> As we know that, CGS, MKS and SI are metric or decimal systems of units. This is because the multiplies and sub-multiplies of their basic units are related to the practical units by powers of 10.

# SOME IMPORTANT PRACTICAL UNITS

## For Length/Distance

(i) **Astronomical Unit** It is the mean distance of the earth from the sun. 1 AU $= 1.496 \times 10^{11}$ m.

(ii) **Light year** It is the distance travelled by light in vacuum in one year. 1 ly $= 9.46 \times 10^{15}$ m

(iii) **Parallactic second** It is the distance at which an arc of length 1 astronomical unit subtends an angle of 1 second of arc. 1 parsec $= 3.084 \times 10^{16}$ m $= 3.26$ ly

(iv) **Micron or micrometer**, $1\ \mu m = 10^{-6}$ m

(v) **Nanometer**, 1 nm $= 10^{-9}$ m

(vi) **Angstrom unit**, $1\ \text{Å} = 10^{-10}$ m

(vii) **Fermi** This unit is used for measuring nuclear sizes 1 Fm $= 10^{-15}$ m

**EXAMPLE |2| Length Conversion**

How many parsec are there in one metre?

***Sol.*** Given, 1 parsec $= 3.084 \times 10^{16}$ m

or $3.084 \times 10^{16}$ m $= 1$ parsec

$$\therefore \quad 1\text{m} = \frac{1}{3.084 \times 10^{16}} \text{ parsec} = 3.25 \times 10^{-17} \text{ parsec}$$

**EXAMPLE |3| Relation between different unit of length**

Deduce relations between astronomical unit, light year and parsec. Arrange them in decreasing order of their magnitudes.

***Sol.*** We know that 1 AU $= 1.5 \times 10^{11}$ m

1 ly $= 9.46 \times 10^{15}$ m

1 par sec $= 3.08 \times 10^{16}$ m

$$\therefore \quad \frac{1 \text{ ly}}{1 \text{ AU}} = \frac{9.46 \times 10^{15}}{1.5 \times 10^{11}} = 6.3 \times 10^4$$

$$1 \text{ ly} = 6.3 \times 10^4 \text{ AU} \quad \text{...(i)}$$

Conversion of light year into parsec

$$\text{i.e.,} \quad \frac{1 \text{ parsec}}{1 \text{ ly}} = \frac{3.08 \times 10^{16}}{9.46 \times 10^{15}} = 3.26$$

$$\therefore \quad 1 \text{ parsec} = 3.26 \text{ ly} \quad \text{...(ii)}$$

Comparing results from Eqs. (i) and (ii), we get

1 parsec > 1 ly > 1 AU

## For Mass

(i) Pound, 1 lb $= 0.4536$ kg

(ii) Slug, 1 Slug $= 14.59$ kg

(iii) Quintal, 1 q $= 100$ kg

(iv) Tonne or Metric ton, 1 t $= 1000$ kg

(v) Atomic mass unit (it is defined as the 1/12th of the mass of one $^{12}_{6}C$ atom) 1 amu $= 1.66 \times 10^{-27}$ kg

## For Time

(i) **Solar day** It is the time taken by the earth to complete one rotation about its own axis w.r.t. the sun.

(ii) **Sedrial year** It is the time taken by the earth to complete one rotation about its own axis w.r.t. a distant star.

(iii) **Solar year** It is the time taken by the earth to complete one revolution around the sun in its orbit.

1 solar year = 365.25 average solar days

= 366.25 sedrial days

(iv) **Tropical year** The year in which there is total solar eclipse is called **tropical year**.

(v) **Leap year** The year which is divisible by 4 and in which the month of February has 29 days is called a **leap year**.

(vi) **Lunar month** It is the time taken by the moon to complete one revolution around the earth in its orbit.

1 lunar month = 27.3 days

(*vii*) **Shake** It is the smallest practical unit of time.

$$1 \text{ shake} = 10^{-8} \text{ s}$$

**EXAMPLE |4| A Clock**

Which type of phenomena can be used as a measure of time? Give three examples.

***Sol.*** A phenomena which repeats itself at regular intervals can be used as a measure of time.

Some examples are given below

(i) oscillation of a pendulum

(ii) rotation of earth around its axis

(iii) revolution of earth around the sun

## For Areas

(i) Barn, $1 \text{ barn} = 10^{-28} \text{ m}^2$

(ii) Acre, $1 \text{ acre} = 4047 \text{ m}^2$

(iii) Hectare, $1 \text{ hectare} = 10^4 \text{ m}^2$

## For Other Quantities

(i) Litre (for volume), $1 \text{ L} = 10^3 \text{ cc} = 10^{-3} \text{m}^3$

Where, cc represents cubic centimetre, i.e. $\text{cm}^3$.

(ii) Gallon (for volume),

In USA, 1 gallon = 3.7854 L

In UK, 1 gallon = 4.546 L

(iii) Pascal (for pressure), $1 \text{ Pa} = 1 \text{ Nm}^{-2}$

Pressure exerted by earth's, atmosphere.

$1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$

(iv) Bar (for pressure),

$1 \text{ bar} = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} = 760 \text{ mm of Hg}$

(v) Torr (for pressure),

1 torr = 1 mm of Hg column = 133.3 Pa

(vi) Electron volt,

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

(vii) Erg (for energy/work),

$1 \text{erg} = 10^{-7} \text{ J}$

(viii) Kilowatt hour (for energy),

$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

(ix) Horse power (for power),

1 HP = 746 W

(x) Dioptre (for power of a lens),

$1 \text{ D} = 1 \text{m}^{-1}$

(xi) Degree (for angle), $1^\circ = \dfrac{\pi}{180} \text{ rad}$

# TOPIC PRACTICE 1

## OBJECTIVE Type Questions

**1.** *A* is the fundamental quantity. Here, *A* refers to

(a) mass
(b) velocity
(c) acceleration
(d) linear momentum

***Sol.*** (a) Mass is the fundamental quantity as it does not depend upon other physical quantities. However, other three quantities, i.e. velocity, acceleration and linear momentum are not fundamental quantities as these show their dependency on fundamental quantities.

**2.** How many wavelength of $Kr^{86}$ are there in 1 m?

(a) 1553164.13 (b) 1650763.73
(c) 2348123.73 (d) 652189.63

***Sol.*** (b) The number of wavelengths of $Kr^{86}$ in 1 m is 1650763.73.

**3.** The solid angle subtended by the periphery of an area $1 \text{cm}^2$ at a point situated symmetrically at a distance of 5 cm from the area is

(a) $2 \times 10^{-2}$ sr (b) $4 \times 10^{-2}$ sr
(c) $6 \times 10^{-2}$ sr (d) $8 \times 10^{-2}$ sr

***Sol.*** (b) Solid angle, $d\Omega = \dfrac{dA}{r^2} = \dfrac{1 \text{ cm}^2}{(5 \text{ cm})^2}$

$= 0.04 \text{ sr}$

$= 4 \times 10^{-2} \text{ sr}$

**4.** Which of the following is not a physical quantity?

(a) Time (b) Impulse
(c) Mass (d) Kilogram

**Sol.** (d) Kilogram represents the unit of a physical quantity. But other three, i.e. time, impulse and mass are the physical quantities.

**5.** The quantity having the same unit in all system of unit is

(a) mass (b) time
(c) length (d) temperature

**Sol.** (b) Time is the quantity which has the same unit in all systems of unit, i.e. second. Other three quantities, i.e. mass, length and temperature have different units in different systems.

**6.** SI unit of capacitance is

(a) ohm-second (b) Wb
(c) coulomb $(\text{volt})^{-1}$ (d) A-$m^2$

**Sol.** (c) SI unit of capacitance is coulomb $(\text{volt})^{-1}$. However, ohm-second is the unit of inductance, Wb is the unit of magnetic flux and A-$m^2$ is the uinit of magnetic moment.

**7.** The damping force on an oscillator is directly proportional to the velocity. The unit of the constant of proportionality is

(a) kg $ms^{-1}$ (b) kg $ms^{-2}$ (c) kg $s^{-1}$ (d) kg s

**Sol.** (c) Given, damping force $\propto$ velocity

$$F \propto v \Rightarrow F = kv$$

$$\Rightarrow \quad k = \frac{F}{v}$$

$$\text{Unit of } k = \frac{\text{Unit of } F}{\text{Unit of } v} = \frac{\text{kg-ms}^{-2}}{\text{ms}^{-1}} = \text{kg s}^{-1}$$

**8.** Number of fermi in one metre is

(a) $10^6$ F (b) $10^{17}$ F (c) $10^{15}$ F (d) $10^{14}$ F

**Sol.** (c) 1 fermi (F) $= 10^{-15}$ m

$$\text{or} \quad 1\text{m} = \frac{1}{10^{-15}} = 10^{15}\text{F}$$

**9.** Number of degrees present in one radian is

(a) 58° (b) 57.3° (c) 56.3° (d) 56°

**Sol.** (b) We know that,

$$\pi \text{ radian} = 180°$$

$$1 \text{ radian} = \frac{180}{\pi} = \frac{180}{22} \times 7 = 57.3°$$

## VERY SHORT ANSWER Type Questions

**10.** Is it possible to have length and velocity both as fundamental quantities? Why?

**Sol.** No, since length is fundamental quantity and velocity is the derived quantity.

**11.** How many astronomical units make one metre?

**Sol.** 1 m = $6.67 \times 10^{-12}$ AU

**12.** How many light years make 1 parsec?

**Sol.** 3.26 light years make 1 parsec.

**13.** Which of these is largest : astronomical unit, light year and parsec?

**Sol.** Parsec is larger than light year which in turn is larger than an astronomical unit.

**14.** Which unit is used to measure size of a nucleus?

**Sol.** The size of nucleus is measured in fermi.

1 fermi $= 10^{-15}$ m

**15.** What is the difference between nm, mN and Nm?

**Sol.** nm stands for nanometre, 1 nm $= 10^{-9}$ m, mN stands for milli-newton, 1 mN $= 10^{-3}$ N, Nm stands for newton metre.

**16.** How many amu make 1 kg?

**Sol.** 1 amu $= 1.66 \times 10^{-27}$ kg

$$\therefore \quad 1 \text{ kg} = \frac{1}{1.66 \times 10^{-27}} = 0.6 \times 10^{27} \text{ amu}$$

**17.** Human heart is an inbuilt clock. Comment.

**Sol.** True, because human heart beats at a regular rate.

**18.** Define one Barn. How it is related with metre?

**Sol.** One barn is a small unit of area used to measure area of nuclear cross-section.

$$\therefore \quad 1 \text{ barn} = 10^{-28} \text{ m}^2$$

## SHORT ANSWER Type Questions

**19.** Express an acceleration of 10 $m/s^2$ in $km/h^2$.

**Sol.**

$$\text{Acceleration} = \frac{10 \text{ m}}{(1\text{s})^2} = \frac{10 \times 10^{-3}}{\left[\frac{1}{60 \times 60}\text{h}\right]^2}$$

$$= (3600)^2 \times 10^{-2} \text{ km/h}^2$$

$$= 1.29 \times 10^5 \text{ km/h}^2$$

**20.** Does AU and Å represent the same unit of length?

**Sol.** No, AU and Å represent two different units of length.

1 AU = 1 astronomical unit $= 1.496 \times 10^{11}$ m

1 Å = 1 angstrom $= 10^{-10}$ m

**21.** Find the value of one light year in giga metre.

**Sol.** We know that,

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

Also, 1 Gm $= 10^9$ m

$$\therefore \quad 1 \text{ ly} = \frac{9.46 \times 10^{15}}{10^9} = 9.46 \times 10^6 \text{ Gm}$$

**22.** **If the velocity of light is taken as the unit of velocity and year as the unit of time, what must be the unit of length? What is it called?**

***Sol.*** Unit of length = Unit of velocity × Unit of time

$$= 3\times10^8\ \text{ms}^{-1} \times 1\ \text{year}$$
$$= 3\times10^8\ \text{ms}^{-1} \times 365\times 24\times 60\times 60\ \text{s}$$
$$= 9.45\times10^{15}\ \text{ms}^{-1} = 1\ \text{ly}$$

**23.** **How many metric tons are there in teragram?**

***Sol.*** In 1 teragram $= 10^{12}$ g

In 1 metric ton $= 10^3\text{kg} = 10^3\times10^3 = 10^6$ g

∴ Number of metric tons are in teragram

$$= \frac{10^{12}\ \text{g}}{10^6\ \text{g}} = 10^6$$

**24.** **What is common between bar and torr?**

***Sol.*** Both bar and torr are the units of pressure.

1 bar = 1 atmospheric pressure
= 760 mm of Hg column
$= 10^5\text{N/m}^2$

1 torr = 1 mm of Hg column

∴ 1 bar = 760 torr

**25.** **The mass of a proton is $1.67\times10^{-27}$ kg. How many protons would make 1 g?**

***Sol.*** Number of protons $= \frac{\text{Total mass}}{\text{Mass of each proton}}$

$$= \frac{10^{-3}}{1.67\times10^{-27}} = 5.99\times10^{23}$$

## LONG ANSWER Type I Questions

**26.** **Why length, mass and time are chosen as base quantities in mechanics?**

***Sol.*** In mechanics, length, mass and time are chosen as the base quantities because

(i) there is nothing simpler to length, mass and time.

(ii) all other quantities in mechanics can be expressed in terms of length, mass and time.

(iii) length, mass and time cannot be derived from one another.

**27.** **Express the average distance of earth from the sun in (i) light year (ii) parsec.**

***Sol.*** Average distance of earth from the sun is $(r)$

i.e. $r = 1\ \text{AU} = 1.496\times10^{11}$ m

$$= \frac{1.496\times10^{11}}{9.46\times10^{15}}\ \text{ly} = 1.58\times10^{-5}\ \text{ly}$$

Also, $r = \frac{1.496\times10^{11}}{3.08\times10^{16}}$ parsec

$$= 4.86\times10^{-6}\ \text{parsec}$$

**28.** **The radius of atom is of the order of 2 Å and radius of a nucleus is of the order of fermi. How many magnitudes higher is the volume of atom as compared to the volume of nucleus?**

***Sol.*** $R_A$, i.e. radius of atom is 2 Å $= 2\times10^{-10}$ m

$R_N$, i.e. radius of nucleus is 1 fermi $= 10^{-15}$ m

$$\frac{V_A}{V_N} = \frac{\frac{4}{3}\pi R_A^3}{\frac{4}{3}\pi R_N^3} = \left[\frac{R_A}{R_N}\right]^3$$
$$= \left[\frac{2\times10^{-10}}{10^{-15}}\right]^3 = 8\times10^{15}$$

**29.** **The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å.**

**1 Å $= 10^{-10}$ m. The size of the hydrogen atom is about 0.5 Å. What is the total atomic volume in $m^3$ of a mole of hydrogen atoms?** **[NCERT]**

***Sol.*** Radius of a hydrogen atom $(r) = 0.5$ Å $= 0.5\times10^{-10}$ m

Volume of each hydrogen atom $(V) = \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times 3.14\times(0.5\times10^{-10})^3$$
$$= 5.234\times10^{-31}\text{m}^3$$

Number of atoms in 1 mole of hydrogen
= Avogadro's number $(N)$
$= 6.023\times10^{23}$

∴ Atomic volume of 1 mole of hydrogen atoms $(V')$
= Volume of a hydrogen atom × Number of atoms

$$V' = V\times N$$
$$= 5.236\times10^{-31}\times6.023\times10^{23}\text{m}^3$$
$$= 3.152\times10^{-7}\ \text{m}$$

**30.** **Why has second been defined in terms of periods of radiations from cesium-133?**

***Sol.*** Second has been defined in terms of periods of radiation, because

(i) this period is accurately defined.

(ii) this period is not affected by change of physical conditions like temperature, pressure and volume etc.

(iii) the unit is easily reproducible in any good laboratory.

## ASSESS YOUR TOPICAL UNDERSTANDING

### OBJECTIVE Type Questions

1. Which one of the following is not a unit of British system of units?
(a) Foot (b) Metre
(c) Pound (d) Second

2. Which of the following statement is incorrect regarding mass?
(a) It is a basic property of matter
(b) The SI unit of mass is candela
(c) The mass of an atom is expressed in u
(d) None of the above

3. Pascal is the unit of
(a) force (b) stress (c) work (d) energy

4. The surface area of a solid cylinder of radius 2.0 cm and height *A* cm is equal to $1.5 \times 10^4$ $(mm)^2$. Here, *A* refers to
(a) 0.9 cm (b) 10 cm
(c) 30 cm (d) 15 cm

5. If the value of force is 100 N and value of acceleration is 0.001 $ms^{-2}$, what is the value of mass in this system of units?
(a) $10^3$ kg (b) $10^4$ kg
(c) $10^5$ kg (d) $10^6$ kg

6. Young's modulus of steel is $1.9 \times 10^{11}$ N/$m^2$. When expressed in CGS units of dyne/$cm^2$, it will be equal to ($1N = 10^5$ dyne, $1 \ m^2 = 10^4 \ cm^2$)
**[NCERT Exemplar]**
(a) $1.9 \times 10^{10}$ (b) $1.9 \times 10^{11}$
(c) $1.9 \times 10^{12}$ (d) $1.9 \times 10^{13}$

7. If the size of bacteria is 1μ, then the number of bacteria in 1 m length will be
(a) one hundred (b) one crore
(c) one thousand (d) one million

8. Among the given following units which one is not unit of length?
(a) Angstrom (b) Fermi
(c) Barn (d) Parsec

9. Age of the universe is about $10^{10}$ yr, whereas the mankind has existed for $10^6$ yr. For how many seconds would the man have existed if age of universe were 1 day?
(a) 9.2 s (b) 10.2 s
(c) 8.6 s (d) 10.5 s

**Answer**

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (b) | 3. (b) | 4. (b) | 5. (c) |
| 6. (c) | 7. (d) | 8. (c) | 9. (c) | |

### VERY SHORT ANSWER Type Questions

10. Explain the concept of mass, length and time which is related to basic or fundamental quantities in mechanics.

11. How do we make the choice of a standard/unit of measurement?

12. Why MKS system had to be rationalised to obtain SI?

13. What is a coherent system of units?

### SHORT ANSWER Type Questions

14. Briefly, explain the measuring process of any physical quantity.

15. Why 'metre' has been defined in terms of wavelength and 'second' in terms of periods of radiation or rotation?

16. Calculate the surface area of a solid cylinder of diameter 4 cm and height 20 cm in $mm^2$.
[**Ans.** 27657.1 $mm^2$]

17. The density of air is 1.293 kg/$m^3$. Express it in CGS units. [**Ans.** 0.001293 g/cc]

18. What is the average distance of earth from the sun?
[**Ans.** $1.496 \times 10^{11}$ m]

19. An Astronomical Unit (AU) is the average distance between earth and the sun, approximately $1.50 \times 10^8$ km. The speed of light is about $3 \times 10^8$ m/s. Express the speed of light in astronomical unit per minute. [**Ans.** 0.12 AU/min]

### LONG ANSWER Type I Questions

20. The radius of gold nucleus is 41.3 fermi. Express its volume in $m^3$. [**Ans.** $2.95 \times 10^{-40}$ $m^3$]

21. Which unit can be used for measuring of very small masses?

22. "Five litres of benzene will weigh more in summer or winter". Comment.

23. Is the measure of an angle dependent upon the unit of length?

**24.** The height of mercury column in a barometer in Calcutta a laboratory was recorded to be 75 cm. Calculate this pressure in SI and CGS units using the following data.

Specific gravity of mercury = 13.6, Density of watch $= 10^3$ kg/m$^3$, $g = 9.8$ m/s$^2$ at Calcutta. Pressure $= h\rho g$ in usual symbols. [**Ans.** $10^{10^4}$ N/m$^2$, $10 \times 10^5$ dyne/cm$^2$]

**25.** The normal duration of ISc Physics practical period in Indian colleges is 100 minutes. Express this period in microcenturies. 1 microcentury $= 10^{-6} \times 100$ years. How many microcenturies did you leep yesterday?

[**Ans.** 1.9 microcenturies]

# |TOPIC 2|

# Significant figures & Rounding off

## SIGNIFICANT FIGURES

Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus first digit that is uncertain. The digits that are known reliably plus the first uncertain digit are known as **significant digits** or **significant figures.**

e.g. When a measured distance is reported to be 374.5 m, it has four significant figures 3, 7, 4 and 5. The figures 3, 7, 4 are certain and reliable, while the digit 5 is uncertain.

Clearly, the digits beyond the significant digits reported in any result are superfluous.

The rules for determining the number of significant figures are

*Rule* 1 All non-zero digits are significant.

e.g. $x = 1234$ has four significant figures.

*Rule* 2 All the zeros between two non-zero digits are significant, no matter where, the decimal point is, if at all.

e.g. $x = 1007$ has four significant figures and $x = 10.07$ also contains four significant figures.

*Rule* 3 If the number is less than one, the zero(s) on the right of decimal point and to the left of first non-zero digit are not significant.

e.g. In $\underline{0}.\underline{0}\underline{0}5704$, the underlined zeros are not significant. The zero between 7 and 4 is significant. The number of significant figures is 4.

*Rule* 4 In a number without a decimal point, the terminal or trailing zeros are not significant.

e.g. $x = 3210$ has three significant figures, the trailing zeros are not significant.

*Rule* 5 The trailing zero(s) in a number with a decimal point are significant.

e.g. 3.500 has four significant figures.

*Rule* 6 The multiplying or dividing factors, which are neither rounded numbers nor numbers representing measured values are exact. They have infinite number of significant digits as per the condition.

e.g. In radius, $r = \dfrac{d}{2}$ and circumference, $s = 2\pi r$, the factors 2 is an exact number. It can be written as 2.0, 2.00 and 2.000, etc. as required.

*Rule* 7 The number of significant figures does not depend on the system of units. So, 16.4 cm, 0.164 m and 0.000164 km, all have three significant figures.

### *Ambiguity in Significant Figures*

There can be some confusion regarding the trailing zeros. Suppose a measured length is reported as $x = 4.700$ m. Clearly, the zeros are meant to convey the precision of measurement and are therefore, significant. If we can rewrite the same length as $x = 0.004700$ km; $x = 470.0$ cm and $x = 4700$ mm.

As per rule 4, we would erroneously conclude that $x = 4700$ mm has two significant digits. While $x = 0.004700$ km has four significant digits and a mere change of units cannot change the number of significant figures (Rule 7).

To remove such ambiguities in determining the number of significant figures, the best way is to report every measurement in scientific notation (in the power of 10).

In this notation, every number can be expressed as

$$a \times 10^b$$

where, $a$ is the number between 1 to 10 and $b$ is any positive or negative exponent (or power) of 10.

Then, the number can be expressed approximately as $10^b$ in which exponent (or power) $b$ of 10 is called order of magnitude of the quantity.

$$x = 4.700 \text{ m} = 4.700 \times 10^{-3} \text{ km}$$

In this case, the number of significant digit is 4, as the power of 10 is irrelevant to the determination of significant figures.

**EXAMPLE |1| Uncertainty in Measurement**

Write down the number of significant figure in the following.

(i) 0.072 (ii) 12.000 (iii) 0.060

(iv) $3.08 \times 10^{11}$ (v) 1.2340 (vi) 0.04

*Sol.* (i) Two (ii) Five (iii) Two
(iv) Three (v) Five (vi) One

# ROUNDING OFF

The result of computation with approximate numbers, which contain more than one uncertain digit, should be rounded off. While rounding off measurements, we use the following rules by convention:

***Rule* 1** If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
e.g. $x = 7.82$ is rounded off to 7.8.

***Rule* 2** If the digit to be dropped is more than 5, then the preceding digit is raised by one.
e.g. $x = 6.87$ is rounded off to 6.9.

***Rule* 3** If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one. e.g. $x = 16.351$ is rounded off to 16.4.

***Rule* 4** If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is left unchanged, if it is even.
e.g. $x = 3.250$ becomes 3.2 on rounding off.

***Rule* 5** If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digits is raised by one, if it is odd.
e.g. $x = 3.750$ is rounded off to 3.8.

**EXAMPLE |2| Precise Value**

Round off the following number as indicated

(i) 18.35 upto 3 digits = 18.4

(ii) 143.45 upto 4 digits = 143.4

(iii) 189.67 upto 3 digits = 190

(iv) 321.1355 upto 5 digits = 321.14

(v) $31.325 \times 10^{-5}$ upto 4 digits $= 31.32 \times 10^{-5}$

(vi) $101.55 \times 10^{6}$ upto 4 digits $= 101.6 \times 10^{6}$

# RULES FOR ARITHMETICAL OPERATIONS WITH SIGNIFICANT FIGURES

Any result is calculated by compounding (i.e. adding/ subtracting/multiplying/dividing) two or more variables, which might have been measured with different degrees of accuracy. Inaccuracy in the measurement of any one variable affects the accuracy of the final result.

Therefore, the result of arithmetical operations performed with measurements cannot be more accurate than the original measurement themselves. The following rules for arithmetic operations with significant figures that make final result more consistent with the precision of the measured values.

## Addition and Subtraction

In both, addition and subtraction, the final result should retain as **many decimal places** as are there in the number with the **least decimal** places. e.g. The sum of three measurement of length 2.1 m, 1.78 m and 2.046 m is 5.926 m, which is rounded off to 5.9 m (upto smallest number of decimal places in the three measurements).

Similarly, if $x = 12.587$ m, and $y = 12.5$ m, then $(x - y)$ is $12.587 - 12.5 = 0.087$ m, which is rounded off to 0.1 m, upto smallest number of decimal places in $y$.

## Multiplication and Division

In multiplication or division, the final result should retain as many significant figures, as are there in the original number with the least significant figures.

e.g. The speed of light is $3.00 \times 10^{8}$ m/s and one year has $3.1557 \times 10^{7}$ s, then light year is given by

$$= 3.00 \times 10^{8} \times 3.155 \times 10^{7}$$
$$= 9.4685 \times 10^{15} \text{ m}$$
$$= 9.5 \times 10^{15} \text{ m}$$

[Rounded off upto 2 significant figures]

**EXAMPLE |3| A Rolling Cube**

Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures? **[NCERT]**

*Sol.* Given, Side of the cube = 7.203 m

Total surface area $= 6 \times (\text{side})^2 = 6 \times (7.203)^2$

$= 311.299254 \text{ m}^2 = 311.3 \text{ m}^2$

[Rounded off to 4 significant figures]

Volume $= (\text{side})^3 = (7.203)^3$

$= 373.714754 \text{ m}^3 = 373.7 \text{ m}^3$.

[Rounded off to 4 significant figures]

**EXAMPLE |4| Density of a Substance**

5.74 g of a substance occupies $1.2 \text{ cm}^3$. Express its density keeping significant figures in view. **[NCERT]**

*Sol.* Density $= \frac{\text{Mass}}{\text{Volume}} = \frac{5.74 \text{ g}}{1.2 \text{ cm}^3}$

$= 4.783 \text{ g cm}^{-3}$

$= 4.8 \text{ g cm}^{-3}$

[Rounded off upto 2 significant figures]

## RULES FOR DETERMINING UNCERTAINTY IN THE RESULTS OF ARITHMETIC CALCULATIONS

***Rule* 1** Suppose, we use a metre scale to measure length and breadth of a thin rectangular sheets as 15.4 cm and 10.2 cm, respectively.

Each measurement has three significant figures and a precision upto first place of decimal. Therefore, we can write length $(l) = (15.4 \pm 0.1)$ cm

$$= 15.4 \pm \left(\frac{0.1}{15.4} \times 100\right) = 15.4 \text{ cm} \pm 0.6\%$$

Similarly, breadth $(b) = (10.2 \pm 0.1)$ cm

$$= 10.2 \pm \left(\frac{0.1 \times 100}{10.2}\right)\% = 10.2 \text{ cm} \pm 1\%$$

Thus, the error of the product i.e. area of thin sheet is $15.4 \times 10.2 \pm (0.6 + 1.0)$ cm$^2$

$$= 157.08 \text{ cm}^2 \pm 1.6\%$$

$$= 157.08 \text{ cm}^2 \pm \left(\frac{1.6}{100} \times 157.08\right) \text{cm}^2$$

$$= (157.08 \pm 2.51) \text{ cm}^2.$$

As per rule, the final value of area can contain only three significant figures and error can contain only one significant figures, we can write the final result as $A = (157 \pm 3) \text{ cm}^2$

***Rule* 2** If a set of experimental data is specified to $n$ significant figures, a result obtained by combining the data will also be valid to $n$ significant figures.

e.g. $x = 13.7$ m and $y = 8.08$ m, both have three significant figures.

Now, $x - y = 13.7 - 8.08 = 5.62$ m

So, the final result should retain as many decimal places as, there is the number with least decimal places. Therefore, rounding off to one place of a decimal, we get $x - y = 5.6$ cm

***Rule* 3** The relative error of a value of number specified to $n$ significant figures depends not only on $n$, but also on the number itself.

e.g. $m_1 = (1.04 \pm 0.01)$ kg

and $m_2 = (9.24 \pm 0.01)$ kg

Relative error in 1.04 kg is

$$\pm\left(\frac{0.01}{1.04}\right) \times 100 = \pm 1\%$$

Similarly, the relative error in 9.24 kg is

$$\pm\left(\frac{0.01}{9.24}\right) \times 100 = \pm 0.1\%$$

Thus, the relative error depends on the number itself.

***Rule* 4** In a multi-step computation, the intermediate results should be calculated to one more significant figure in every measurement, then the number of digits in the least precise measurement.

e.g. $x = 9.58$ has three significant digits. Now, reciprocal of $x$ is $\frac{1}{x} = \frac{1}{9.58} = 0.104$, rounded off to three significant digits. When we take reciprocal of 0.104, we get 9.62, rounded off to three significant digits.

However, if we calculate $\frac{1}{x} = \frac{1}{9.58} = 0.1044$, rounded off to four significant figures, then

$\frac{1}{0.1044} = 9.58$, rounded off to three significant digits.

Thus, retaining one more extra digits in intermediate steps of complex calculations would avoid additional errors in the process of rounding off the numbers.

**EXAMPLE |5| Appropriate Numbers of Significant Number**

Solve the following and express the result to an appropriate number of significant figures.

(i) Add 6.2g, 4.33g and 17.456g

(ii) Subtract 63.54 kg from 187.2 kg

(iii) $75.5 \times 125.2 \times 0.51$

(iv) $\frac{2.13 \times 24.78}{458.2}$

*Sol.* (i) $6.2 \text{ g} + 4.33 \text{ g} + 17.456 \text{ g} = 27.986 = 28.0 \text{ g}$

[rounded off to first decimal place]

(ii) $187.2 \text{kg} - 63.54 \text{kg} = 123.66 \text{kg} = 123.7 \text{kg}$

[rounded off to first decimal place]

(iii) $75.5 \times 125.2 \times 0.51 = 4820.826 = 4800$

[rounded off upto two significant figures]

(iv) $\frac{2.13 \times 24.78}{458.2} = 0.115193 = 0.115$

[rounded off to three significant figures]

**EXAMPLE |6|**

The length, breadth and height of a rectangular block of wood were measured to be

$l = 12.13 \pm 0.02$ cm, $b = 8.16 \pm 0.01$ cm

and $h = 3.46 \pm 0.01$ cm

Determine the percentage error in the volume of the block upto correct significant figures.

**Sol.** Volume of block, $V = lbh$

The percentage error in the volume is given by

$$\frac{\Delta V}{V} \times 100 = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}\right) \times 100$$

$$= \left(\frac{0.02}{12.13} + \frac{0.01}{8.16} + \frac{0.01}{3.46}\right) \times 100$$

$$= \frac{200}{1213} + \frac{100}{816} + \frac{100}{346}$$

$$= 0.1649 + 0.1225 + 0.2890$$

$$= 0.58\%$$

[rounded off to two significant figures]

# TOPIC PRACTICE 2

## OBJECTIVE Type Questions

**1.** The ratio of the volume of the atom to the volume of the nucleus is of the order of

(a) $10^{15}$ (b) $10^{25}$ (c) $10^{20}$ (d) $10^{10}$

**Sol.** (a) Radius of atom $= 10^{-10}$ m

Radius of nucleus $= 10^{-15}$ m

$$\text{Ratio} = \frac{10^{-10}}{10^{-15}} = 10^5$$

Ratio of volume $= (10^5)^3 = 10^{15}$

**2.** The number of significant figures in the numbers $4.8000 \times 10^4$ and 48000.50 are respectively,

(a) 5 and 6 (b) 5 and 7 (c) 2 and 7 (d) 2 and 6

**Sol.** (b) $4.8000 \times 10^4$ has 4, 8, 0, 0, 0 $\Rightarrow$ 5 significant digits.

48000.50 has 4, 8, 0, 0, 0, 5, 0 $\Rightarrow$ 7 significant digits.

**3.** If $3.8 \times 10^{-6}$ is added to $4.2 \times 10^{-5}$ giving due regard to significant figures, then the result will be

(a) $4.58 \times 10^{-5}$ (b) $4.6 \times 10^{-5}$

(c) $45 \times 10^{-5}$ (d) None of these

**Sol.** (b) By adding $3.8 \times 10^{-6}$ and $42 \times 10^{-6}$, we get

$$= 45.8 \times 10^{-6} = 4.58 \times 10^{-5}$$

As least number of significant figures in given values are 2, so we round off the result to $4.6 \times 10^{-5}$.

**4.** The mass and volume of a body are 4.237 g and 2.5 cm$^3$, respectively. The density of the material of the body in correct significant figures is **[NCERT Exemplar]**

(a) 1.6048 g cm$^{-3}$ (b) 1.69 g cm$^{-3}$

(c) 1.7 g cm$^{-3}$ (d) 1.695 g cm$^{-3}$

**Sol.** (c) In this question, density should be reported to two significant figures.

$$\text{Density} = \frac{4.237\text{g}}{2.5\text{ cm}^3} = 1.6948$$

As rounding off the number, we get density = 1.7

## VERY SHORT ANSWER Type Questions

**5.** If all measurements in an experiment are taken upto same number of significant figures, then which measurement is responsible for maximum error?

**Sol.** The maximum error will be due to

(i) measurement which is least accurate.

(ii) measurement of the quantity which has maximum power in the formula.

**6.** Round off to four significant figures

(i) 36.879 (ii) 1.0084

**Sol.** (i) 36.88 (ii) 1.008

**7.** In a number without decimal, what is the significance of zeros on the right of non-zero digits?

**Sol.** All such zeros are not significant. e.g. $x = 678000$ has only three significant figures.

**8.** Solve with due regard to significant figures $\sqrt{6.5 - 6.32}$

**Sol.** $\sqrt{6.5 - 6.32} = \sqrt{0.18} = \sqrt{0.4242}$, upto one decimal place $= 0.43$ (having 2 significant figures).

## SHORT ANSWER Type Questions

**9.** A jeweller put a diamond weighing 5.42 g in a box weighing 1.2 kg. Find the total weight of the box and the diamond to correct number of significant figures.

**Sol.** Weight of diamond = 5.42 g = 0.00542 kg

Total weight = 1.2 + 0.00542

= 1.20542 kg = 1.2 kg

**10.** The voltage across a lamp is $V = (6.0 \pm 0.1)$ volt and the current passing through it $I = (4 \pm 0.2)$ ampere. Find the power consumed by the electric lamp upto correct significant figures. Given that power, $P = VI$

**Sol.** As, $V = (6.0 \pm 0.1)$ V, $I = (4.0 \pm 0.2)$A

Power, $P = VI = 6.0 \times 4.0 = 24$ W

and maximum error in power measurement

$$\frac{\Delta P}{P} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.1}{6.0} + \frac{0.2}{4.0}$$

$$= 0.017 + 0.050 = 0.067$$

$$\Delta P = 0.067 \times P$$

$$= 0.0\,67 \times 24 = 1.6 \text{ W}$$

Power consumed by the electric lamp within error limit is $(24 \pm 1.6)$ W.

## LONG ANSWER Type I Questions

**11.** The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (i) the total mass of the box (ii) the difference in the mass of the pieces to correct significant figures? **[NCERT]**

**Sol.** Given, mass of the box $(m) = 2.3$ kg

Mass of first gold piece $(m_1) = 20.15$ g $= 0.02015$ kg

Mass of second gold piece $(m_2) = 20.17$ g $= 0.02017$ kg

(i) Total mass of the box $(M) = m + m_1 + m_2$

$$= 2.3 + 0.02015 + 0.02017$$

$$= 2.34032 \text{ kg}$$

As the mass of the box has least decimal place i.e. one decimal place, therefore, total mass of the box can have only one decimal place. Rounding off the total mass of the box up to one decimal place, we get

Total mass of the box $(M) = 2.3$ kg

(ii) Difference in masses of gold pieces

$$(\Delta m) = m_2 - m_1$$

$$= 20.17 - 20.15 = 0.02 \text{ g}$$

(The masses of two gold pieces has two decimal places, therefore, it is correct up to two places of decimal.)

**12.** A physical quantity $P$ is related to four observables $a$, $b$, $c$ and $d$ are as follows $P = a^3 b^2 / \sqrt{c}d$

The percentage errors of measurement in $a$, $b$, $c$ and $d$ are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity $P$? If the value of $P$ calculated using the above relation turns out to be 3.763, to what value should you round off the result? **[NCERT]**

**Sol.** Given, $P = a^3 b^2 / \sqrt{c}d$

Maximum relative error in physical quantity $P$ is given by

$$\frac{\Delta P}{P} = \pm \left[3\left(\frac{\Delta a}{a}\right) + 2\left(\frac{\Delta b}{b}\right) + \frac{1}{2}\left(\frac{\Delta c}{c}\right) + \left(\frac{\Delta d}{d}\right)\right]$$

$\therefore$ Maximum percentage error in $P$ is given by

$$\frac{\Delta P}{P} \times 100 = \pm\left[3\left(\frac{\Delta a}{a} \times 100\right) + 2\left(\frac{\Delta b}{b} \times 100\right) + \frac{1}{2}\left(\frac{\Delta c}{c} \times 100\right) + \left(\frac{\Delta d}{d} \times 100\right)\right]$$

Given $\frac{\Delta a}{a} \times 100 = 1\%$, $\frac{\Delta b}{b} \times 100 = 3\%$

$\frac{\Delta c}{c} \times 100 = 4\%$, $\frac{\Delta d}{d} \times 100 = 2\%$

$$\therefore \frac{\Delta P}{P} \times 100 = \pm\left[3 \times (1) + 2 \times (3) + \frac{1}{2} \times (4) + (2)\right]$$

$$= \pm [3 + 6 + 2 + 2]\% = \pm 13\%$$

As the result (13%) has two significant figures, therefore, the value of $P = 3.763$ should have only two significant figures. Rounding off the value of $P$ up to two significant figures, we get $P = 3.8$

**13.** State the number of significant figures in the following **[NCERT]**

(i) $0.007 \text{ m}^2$ (ii) $2.64 \times 10^{24}$ kg

(iii) $0.2370 \text{ g/cm}^3$ (iv) 6.320 J

(v) $6.032 \text{ N/m}^2$ (vi) $0.0006032 \text{ m}^2$

**Sol.** The number of significant figures in the given quantities are given below.

(i) In 0.007, the number of significant figures is 1 because in a number less than 1, the zero's on the right of the decimal point but to the let of the first non-zero digit are not significant.

(ii) In $2.64 \times 10^{24}$, the number of significant figures is 3 because all non-zero digits are significant, power of 10 are not taken in significant figure.

(iii) In 0.2370, the number of significant figures is 4, as all non-zero digits left to decimal and trailing zero are significant.

(iv) In 6.320, the number of significant figures is 4 (reason is same as in part 'iii').

(v) In 6.032, the number of significant figures is 4 (reason is same as in part 'iii').

(vi) In 0.0006032, the number of significant figures is 4 (reason is same as in part 'i').

**14.** Write down the number of significant figures in the following

(i) 5238 N (ii) 4200 kg

(iii) 34.000 m (iv) 0.02340 N/m

**Sol.** (i) 5238 N has four significant digits.

(ii) 4200 kg $= 4.200 \times 10^3$ kg has four significant figures

(iii) 34.000 m has five significant digits.

(iv) 0.02340 N/m has four significant digits.

**15.** Compute the following with regards to significant figures.

(i) $4.6 \times 0.128$ (ii) $\frac{0.9995 \times 1.53}{1.592}$

(iii) $876 + 0.4382$

**Sol.** (i) $4.6 \times 0.128 = 0.5888 = 0.59$

The result has been rounded off to have two significant digits (as in 4.6)

(ii) $\frac{0.9995 \times 1.53}{1.592} = 0.96057 = 0.961$

The result has been rounded off to three significant digits (as in 1.53).

(iii) $876 + 0.4382 = 876.4382 = 876$

As, there is no decimal point in 876, therefore, result of addition has been rounded off to no decimal point.

**16.** **The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm, respectively. Give the area and volume of the sheet to correct significant figures.** **[NCERT]**

If different values of a same quantity are given in different units, then first of all convert them in same units without changing the number of significant figures.

**Sol.** Given, length $(l) = 4.234$ m, Breadth $(b) = 1.005$ m

$$\text{Thickness } (t) = 2.01 \text{ cm} = 0.0201 \text{ m}$$

$$\text{Area of sheet } (A) = 2(l \times b + b \times t + t \times l)$$
$$= 2[(4.234 \times 1.005) + (1.005 \times 0.0201) + (0.0201 \times 4.234)]$$
$$= 2 \times 4.3604739 = 8.7209478 \text{ m}^2$$

As, thickness has least number of significant figures 3, therefore, rounding off area up to three significant figures, we get Area of sheet $(A) = 8.72 \text{ m}^2$

$$\text{Volume of sheet } (V) = l \times b \times t$$
$$= 4.234 \times 1.005 \times 0.0201$$
$$= 0.0855289$$

Rounding off up to three significant figures, we get

Volume of the sheet $= 0.0855 \text{ m}^3$

## LONG ANSWER Type II Questions

**17.** **The sun is a hot plasma (ionised matter) with its inner core at a temperature exceeding $10^7$ K and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data.** **[NCERT]**

**Mass of the Sun $= 2.0 \times 10^{30}$ kg**

**and radius of the Sun $= 7.0 \times 10^8$ m.**

**Sol.** Given, mass of the sun $(M) = 2.0 \times 10^{30}$ kg

Radius of the Sun $(R) = 7.0 \times 10^8$ m

$$\text{Density of the Sun} = \frac{\text{Mass of the Sun } (M)}{\text{Volume of the Sun } (V)}$$

$$\left[\because \text{Density} = \frac{\text{Mass}}{\text{Volume}}\right]$$

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3}{4}\frac{M}{\pi R^3} = \frac{3 \times 2.0 \times 10^{30}}{4 \times 3.14 \times (7.0 \times 10^8)^3}$$

$$= \frac{3 \times 10^{30}}{6.28 \times 343 \times 10^{24}} = 1.392 \times 10^3$$

$$\rho = 1.4 \times 10^3 \text{ kg/m}^3$$

This density is of the order of density of solids and liquids and not of gases.

The temperature of inner core of the sun is $10^7$ K while the temperature of the outer layers is nearly 6000 K. At so high temperature, no matter can exist in its solid or liquid state. Every matter is highly ionised and present as a mixture of nucleus, free electrons and ions which is called plasma. The density of plasma is so high due to inward gravitational attraction on outer layers due to inner layers of the sun.

**18.** **Estimate the average mass density of sodium atom assuming its size to be about 2.5 Å (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase 970 kg/m$^3$. Are the two densities of the same order of magnitude? If so, why?** **[NCERT]**

**Sol.** Given, radius of sodium atom,

$$r = 2.5 \text{ Å} = 2.5 \times 10^{-10} \text{ m} \qquad [\because 1 \text{ Å} = 10^{-10} \text{ m}]$$

$$\text{Volume of sodium atom} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times 3.14 \times (2.5 \times 10^{-10})^3 = 65.42 \times 10^{-30} \text{ m}^3$$

Number of atom in one mole of sodium

$$= \text{Avogadro's number } (N)$$
$$N = 6.023 \times 10^{23}$$

$\therefore$ Atomic volume of sodium

$$= \text{Volume of one atom of sodium} \times \text{Number of atoms}$$
$$= 65.42 \times 10^{-30} \times 6.023 \times 10^{23} = 3.94 \times 10^{-5} \text{ m}^3$$

Mass of a mole of sodium $= 23 \text{ g} = 23 \times 10^{-3}$ kg

$$\therefore \text{Average mass density of sodium} = \frac{\text{Mass}}{\text{Volume}}$$

$$\rho = \frac{23 \times 10^{-3}}{3.94 \times 10^{-5}} \simeq 5.84 \times 10^2 \text{ kg/m}^3 \simeq 584 \text{ kg/m}^3$$

Density of sodium in crystalline phase

$$= 970 \text{ kg/m}^3 = 9.7 \times 10^2 \text{ kg/m}^3$$

The two densities are of the same order of magnitude because in solid state, atoms are tightly packed.

## ASSESS YOUR TOPICAL UNDERSTANDING

### OBJECTIVE Type Questions

1. Size of the universe is of the order of
   (a) $10^{40}$ m (b) $10^{26}$ m (c) $10^{18}$ m (d) $10^{14}$ m
2. To determine the number of significant figures, scientific notation is
   (a) $a^b$ (b) $a \times 10^b$ (c) $a \times 10^2$ (d) $a \times 10^4$
3. In 4700 m, significant digits are
   (a) 2 (b) 3 (c) 4 (d) 5
4. The number of significant figures in 0.06900 is **[NCERT Exemplar]**
   (a) 5 (b) 4 (c) 2 (d) 3
5. The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is **[NCERT Exemplar]**
   (a) 663.821 (b) 664 (c) 663.8 (d) 663.82
6. Choose the correct option.
   (a) $3.00 - 2.5 = -5.0$ (b) $3.00 - 2.5 = 0.50$
   (c) $3.00 + 2.5 = 5.50$ (d) $3.00 + 2.5 = 5.500$

**Answer**

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (b) | 3. (a) | 4. (b) | 5. (b) |
| 6. (b) | | | | |

### VERY SHORT ANSWER Type Question

7. What do you mean by order of magnitude of a length?

### LONG ANSWER Type II Questions

8. What is the difference between 5.0 and 5.00?
9. What is meant by significant figures? Give any four rules for counting significant figures.

# |TOPIC 3|
## Dimensions of a Physical Quantity

The dimensions of a physical quantity are the powers (or exponents) to which the units of base quantities are raised to represent a derived unit of that quantity. There are seven base quantities and are represented with square brackets [ ] such as length [L], mass [M], time [T], electric current [A], thermodynamic temperature [K], luminous intensity [cd] and amount of substances [mol].

e.g. The volume occupied by an object is expressed as the product of length. So, its dimension is given by

$$V = [\text{L}] \times [\text{L}] \times [\text{L}] = [\text{L}^3]$$

As there is no mass and time in volume, so the dimension of volume is expressed as

$$V = [\text{M}^0\text{L}^3\text{T}^0]$$

Similarly, for force, it is the product of mass and acceleration. It can be expressed as

$$F = \text{mass} \times \text{acceleration} = \text{mass} \times \frac{\text{length}}{(\text{time})^2}$$

$\therefore$ The dimension of force is given by

$$F = [\text{M}] \times \frac{[\text{L}]}{[\text{T}]^2} = [\text{MLT}^{-2}]$$

**Note**

Using the square bracket [ ] round a quantity means that we are dealing with the '**dimensions of**' the quantity.

## DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

The expression which shows how and which of the fundamental quantities represent the dimension of the physical quantity is called the **dimensional formula** of the given physical quantity.

e.g. Some of the dimensional formulae are as given below

$$\text{Acceleration} = [\text{M}^0\text{L}^1\text{T}^{-2}]$$

$$\text{Mass density} = [\text{ML}^{-3}\text{T}^0]$$

$$\text{Volume} = [\text{M}^0\text{L}^3\text{T}^0]$$

The equation obtained by equating a physical quantity with its dimensional formula is called the **dimensional equation** of the given physical quantity.

e.g. Some of the dimensional equations are as given below

$$\text{Linear momentum} = \text{mass} \times \text{velocity} = [\text{M}^1\text{L}^1\text{T}^{-1}]$$

$$\text{Impulse} = \text{Force} \times \text{time} = [\text{M}^1\text{L}^1\text{T}^{-1}]$$

$$\text{Moment of Inertia} = \text{mass} \times \text{radius of gyration} = [\text{M}^1\text{L}^2\text{T}^0]$$

**Problem Solving Strategy**

(Finding Dimensional Formulae)

1. First read the problem carefully and then find out whether we have given with the formulae or any law to describe in it.
2. Write the formulae of a physical quantity for which the dimensions to be known.
3. Convert the formulae of derived physical quantity into fundamental quantities, e.g. acceleration= Length/Time
4. Write the corresponding symbols for fundamental quantities, e.g. mass = [M], length = [L],
   time = [T], etc.
5. Make proper algebraic combination and get the result.
6. Try to arrange the dimensions in order i.e. [M], [L], [T] .....

**EXAMPLE |1| Dimension of Gravitational Constant**

Find out the dimensions of universal gravitational constant used in Newton's law of gravitation.

***Sol.*** According to Newton's law of gravitation, the force $F$, between two masses $m_1$ and $m_2$ separated by distance $r$ can be given as $F = G\frac{m_1 m_2}{r^2}$

Where, $G$ = universal gravitational constant

$$G = \frac{Fr^2}{m_1 m_2} = \frac{\text{Newton} \times (\text{metre})^2}{(\text{kg})^2}$$

$$G = \frac{(\text{mass } \times \text{acceleration}) \times (\text{metre})^2}{(\text{mass})^2}$$

$$= \frac{1}{\text{mass}}\left(\frac{\text{Change in velocity}}{\text{Time}}\right) \times (\text{Length})^2$$

$$G = \frac{(\text{Length})^2}{\text{Mass} \times \text{Time}} \times \frac{\text{Distance}}{\text{Time}}$$

$$G = \frac{[\text{L}]^2}{[\text{M}] \times [\text{T}]} \times \frac{[\text{L}]}{[\text{T}]} = [\text{M}^{-1}\ \text{L}^3\ \text{T}^{-2}]$$

**Dimensional Formula of Some of the Important Mechanical Quantities**

| Physical quantity | Relation with other quantities | Dimensional formula | SI unit |
|---|---|---|---|
| Area | length × breadth | $L \times L = L^2 = [M^0\ L^2\ T^0]$ | $m^2$ |
| Volume | length × breadth × height | $L \times L \times L = L^3 = [M^0\ L^3\ T^0]$ | $m^3$ |
| Density | $\frac{\text{mass}}{\text{volume}}$ | $\frac{M}{L^3} = [M^1\ L^{-3}\ T^0]$ | $kg\ m^{-3}$ |
| Specific gravity | $\frac{\text{density of body}}{\text{density of water at 4°C}}$ | $\frac{M/L^3}{M/L^3} = 1 = [M^0\ L^0\ T^0] \rightarrow$ no dimensions | No units |
| Speed or velocity | $\frac{\text{distance or displacement}}{\text{time}}$ | $\frac{L}{T} = LT^{-1} = [M^0\ L^1\ T^{-1}]$ | $ms^{-1}$ |
| Linear momentum | mass × velocity | $M \times LT^{-1} = [M^1\ L^1\ T^{-1}]$ | $kg\ ms^{-1}$ |
| Acceleration | $\frac{\text{change in velocity}}{\text{time taken}}$ | $\frac{L/T}{T} = LT^{-2} = [M^0\ L^1\ T^{-2}]$ | $ms^{-2}$ |
| Acceleration due to gravity ($g$) | $\frac{\text{change in velocity}}{\text{time taken}}$ | $\frac{L/T}{T} = LT^{-2} = [M^0\ L^1\ T^{-2}]$ | $ms^{-2}$ |
| Force | mass × acceleration | $M \times LT^{-2} = [M^1 L^1 T^{-2}]$ | N (newton) |
| Impulse | force × time | $M\ LT^{-2} \times T = [M^1\ L^1\ T^{-1}]$ | Ns |
| Pressure | force/area | $\frac{MLT^{-2}}{L^2} = [M^1\ L^{-1}\ T^{-2}]$ | $Nm^{-2}$ |
| Universal constant of gravitation ($G$) | From Newton's law of gravitation. $F = \frac{Gm_1m_2}{r^2}$ or $G = \frac{Fr^2}{m_1 m_2}$, where $F$ is force between masses $m_1, m_2$ at a distance $r$ | $G = \frac{[MLT^{-2}]\ L^2}{MM} = [M^{-1}\ L^3\ T^{-2}]$ | $Nm^2\ kg^{-2}$ |
| Work | force × distance | $M\ LT^{-2} \times L = [M^1\ L^2\ T^{-2}]$ | J (joule) |
| Energy (All types) | work | $[M^1\ L^2\ T^{-2}]$ | J (joule) |

| Physical quantity | Relation with other quantities | Dimensional formula | SI unit |
|---|---|---|---|
| Moment of force | force × distance | $M LT^{-2} \times L = [M^1 L^2 T^{-2}]$ | N-m |
| Power | $\frac{\text{work}}{\text{time}}$ | $\frac{M L^2 T^{-2}}{T} = [M^1 L^2 T^{-3}]$ | W (watt) |
| Surface tension | $\frac{\text{force}}{\text{length}}$ | $\frac{M L T^{-2}}{T} = [M^1 L^0 T^{-2}]$ | $Nm^{-1}$ |
| Surface energy | Energy of free surface | $[M^1 L^2 T^{-2}]$ | J |
| Force constant | $\frac{\text{force}}{\text{displacement}}$ | $\frac{M L T^{-2}}{T} = [M^1 L^0 T^{-2}]$ | $Nm^{-1}$ |
| Thrust | force | $[M^1 L^1 T^{-2}]$ | N (newton) |
| Tension | force | $[M^1 L^1 T^{-2}]$ | N (newton) |
| Stress, Pressure | $\frac{\text{force}}{\text{area}}$ | $\frac{M LT^{-2}}{L^2} = [M^1 L^{-1} T^{-2}]$ | $Nm^{-2}$ |
| Strain | $\frac{\text{change in dimension}}{\text{original dimension}}$ | $\frac{L}{L} = [M^0 L^0 T^0]$ | No units |
| Coefficient of elasticity | $\frac{\text{stress}}{\text{strain}}$ | $\frac{M^1 L^{-1}T^{-2}}{1} = [M^1 L^{-1} T^{-2}]$ | $Nm^{-2}$ |
| Radius of gyration (*K*) | distance | $L = [M^0 L^1 T^0]$ | m |
| Moment of inertia (*I*) | mass (radius of gyration)$^2$ | $ML^2 = [M^1 L^2 T^0]$ | kg $m^2$ |
| Angle (θ) or Angular displacement (θ) | $\frac{\text{length } (l)}{\text{radius}(r)}$ | $\frac{L}{L} = 1 = [M^0 L^0 T^0]$ | radian |
| Angular velocity (ω) | $\frac{\text{angle}(\theta)}{\text{time}(t)}$ | $\frac{1}{T} = T^{-1} = [M^0 L^0 T^{-1}]$ | rad $s^{-1}$ |
| Angular acceleration (α) | $\frac{\text{change in angular velocity}}{\text{time taken}}$ | $\frac{1/T}{T} = T^{-2} = [M^0 L^0 T^{-2}]$ | rad $s^{-2}$ |
| Angular momentum | $I\omega$ | $[ML^2]\,[T^{-1}] = [M^1 L^2 T^{-1}]$ | kg $m^2$ $s^{-1}$ |
| Torque | $I\alpha$ | $[ML^2]\,[T^{-2}] = [M^1 L^2 T^{-2}]$ | N-m |
| Wavelength (λ) | length of one wave, i.e., distance | $L = [M^0 L^1 T^0]$ | m |
| Frequency (ν) | number of vibrations/sec | $\frac{1}{T} = T^{-1} = [M^0 L^0 T^{-1}]$ | $s^{-1}$ or Hz (hertz) |
| Angular frequency (ω) | $2\pi \times$ frequency | $T^{-1} = [M^0 L^0 T^{-1}]$ | radian/sec |
| Velocity of light in vacuum (*c*) | $\frac{\text{distance travelled}}{\text{time taken}}$ | $\frac{L}{T} = [M^0 L^1 T^{-1}]$ | $ms^{-1}$ |
| Velocity gradient | $\frac{\text{velocity}}{\text{distance}}$ | $\frac{LT^{-1}}{T} = T^{-1} = [M^0 L^0 T^{-1}]$ | $s^{-1}$ |
| Rate of flow | $\frac{\text{volume}}{\text{time}}$ | $\frac{L^3}{T} = L^3T^{-1} = [M^0 L^3 T^{-1}]$ | $m^3s^{-1}$ |
| Planck's constant (*h*) | $\frac{\text{energy } (E)}{\text{frequency } (\nu)}$ | $\frac{ML^2T^{-2}}{T^{-1}} = [M^1 L^2 T^{-1}]$ | J-s |
| Linear mass density (*m*) | $\frac{\text{mass}}{\text{length}}$ | $\frac{M}{L} = [M^1 L^{-1} T^0]$ | kg $m^{-1}$ |

| Physical quantity | Relation with other quantities | Dimensional formula | SI unit |
|---|---|---|---|
| Distance travelled in *n*th second | $\frac{\text{distance}}{\text{time}}$ | $\frac{L}{T} = [M^0 L^1 T^{-1}]$ | $ms^{-1}$ |
| Avogadro's number (*N*) | Number of atoms/ molecules in one gram atom/mole | $[M^0 L^0 T^0]$ | $mole^{-1}$ |
| Magnetic dipole moment (*M*) | $M = IA$ | $AL^2 = [M^0 L^2 T^0 A^1]$ | $Am^2$ |
| Pole strength (*m*) | $m = \frac{M}{2l}$ | $\frac{AL^2}{L} = AL = [M^0 L^1 T^0 A^1]$ | Am |
| Magnetic permeability of free space ($\mu_0$) | From Coulomb's law in magnetism, $F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$, where $m_1, m_2$ are strengths of two poles; $[\mu_0] = \frac{4\pi [F][r^2]}{m_1 m_2}$ | $\frac{[MLT^{-2}][L^2]}{[AL]^2} = [M^1 L^1 T^{-2} A^{-2}]$ | $Hm^{-1}$ |
| Resistance (*R*) | $\frac{\text{potential difference}}{\text{current}}$ | $\frac{ML^2T^{-3}A^{-1}}{A} = [M^1 L^2 T^{-3} A^{-2}]$ | $\Omega$ (ohm) |
| Capacitance (*C*) | $\frac{\text{charge}}{\text{potential difference}}$ | $\frac{AT}{M L^2 T^{-3} A^{-1}} = [M^{-1} L^{-2} T^4 A^2]$ | F (farad) |
| Surface density of charge | $\sigma = \frac{\text{charge}}{\text{area}}$ | $\frac{AT}{L^2} = [M^0 L^{-2} T^1 A^1]$ | $Cm^{-1}$ |
| Electric dipole moment (*p*) | $q(2a)$ | $AT(L) = [M^0 L^1 T^1 A^1]$ | Cm |
| Specific Resistance or resistivity ($\rho$) | $\frac{Ra}{l}$ | $\frac{[ML^2T^{-3}A^{-2}][L^2]}{L} = [M^1 L^3 T^{-3} A^{-2}]$ | ohm-m |
| Conductance (*G*) | $\frac{1}{R}$ | $\frac{1}{[ML^2T^{-3}A^{-2}]} = [M^{-1} L^{-2} T^3 A^2]$ | $ohm^{-1}$ |
| Conductivity ($\sigma$) | $\frac{1}{\rho}$ | $\frac{1}{[M^1L^3T^{-3}A^{-2}]} = [M^{-1} L^{-3} T^3 A^2]$ | $ohm^{-1}$ |
| Electric flux | Electric field × area | $[M^1L^1T^{-3}A^{-1}][L^2] = [M^1 L^3 T^{-3} A^{-1}]$ | $Nm^2 C^{-1}$ |
| Faraday constant | Avogadro number × elementary charge | $\frac{1}{mol} \times AT = [M^0 L^0 T^1 A^1 mol^{-1}]$ | C |
| Mass defect ($\Delta m$) | Sum of masses of nucleons – mass of nucleus | $[M^1 L^0 T^0]$ | kg |
| Binding energy of nucleus | (mass defect) × (speed of light)$^2$ | $M[LT^{-1}]^2 = [ML^2T^{-2}]$ | J |

**EXAMPLE |2| Pull Over Buddy**

Derive the dimensions formula of physical quantities.

(i) Tension

(ii) Velocity gradient

(iii) Linear mass density

(iv) Impulse

*Sol.* (i) Tension = force = mass × acceleration

$[M] \times [LT^{-2}] = [MLT^{-2}]$

(ii) Velocity gradient $= \frac{\text{Velocity}}{\text{Distance}} = \frac{[LT^{-1}]}{[L]} = [T^{-1}]$

(iii) Linear mass density $= \frac{\text{Mass}}{\text{Length}} = \frac{[M]}{[L]} = [ML^{-1}]$

(iv) Impulse = force × time $= [MLT^{-2}] \times [T] = [MLT^{-1}]$

# DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

The dimensional analysis helps us in deducing the relations among different physical quantities and checking the accuracy, derivation and dimensional consistency or its homogeneity of various numerical expressions.

Its applications are as given below

1. Checking the dimensional consistency of equations
2. Conversion of one system of units into another
3. Deducing relation among the physical quantities

## 1. Checking the Dimensional Consistency of Equations

The magnitudes of physical quantities may be added together or subtracted from one another only if they have the same dimensions.

Thus, mass cannot be added to velocity or an electric current cannot be subtracted from time. We use the principle of homogeneity of dimensions to check the consistency and correctness of an equation.

The **principle of homogeneity of dimension** states that a physical quantity equation will be dimensionally correct, if the dimensions of all the terms occurring on both sides of the equation are same.

e.g. Let us check the dimensional consistency of the equation of motion as

$$s = ut + \frac{1}{2}at^2$$

Dimensions of different terms are

$$[s] = [L]$$

$$[ut] = [LT^{-1}][T] = [L]$$

$$\left[\frac{1}{2}at^2\right] = [LT^{-2}]/[T^2] = [L]$$

As all the terms on both sides of the equations have the same dimensions, so the given equation is dimensionally correct.

**EXAMPLE |3| Test of Consistency**

Check whether the given equation is dimensionally correct $\frac{1}{2}mv^2 = mgh$. **[NCERT]**

*Sol.* The dimensions of LHS

$= [M]\,[LT^{-1}]^2 = [ML^2T^{-2}]$

The dimensions of RHS $= [M]\,[LT^{-2}][L] = [ML^2T^{-2}]$

The dimensions of LHS and RHS are same and hence the consistency is verified.

**EXAMPLE |4| Analysis of an Equation**

Check the dimensional consistency of the following equations.

(i) de-Broglie wavelength, $\lambda = \frac{h}{mv}$

(ii) Escape velocity, $v = \sqrt{\frac{2GM}{R}}$

*Sol.* (i) Given, $\lambda = \frac{h}{mv}$

LHS as wavelength is a distance $\lambda = [L]$

Also RHS, $\frac{h}{mv} = \frac{\text{Planck's constant}}{\text{Mass} \times \text{Velocity}}$

$= \frac{[ML^2T^{-1}]}{[M] \times [LT^{-1}]} = [L]$

$\therefore$ LHS = RHS

Hence, the given equation is dimensionally correct.

(ii) Here, $v = \sqrt{\frac{2GM}{R}}$

LHS $v = [LT^{-1}]$ RHS $= \left[\frac{2GM}{R}\right]^{1/2}$

$G = [M^{-1}L^3T^{-2}]$, $R = [L]$, $M = [M]$

$= \left[\frac{M^{-1}L^3T^{-2}M}{L}\right]^{1/2} = [L^2T^{-2}]^{1/2} = [LT^{-1}]$

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence, the equation is dimensionally correct.

## 2. Conversion of One System of Units into Another

As we know numerical value is inversely proportional to the size of the unit but the magnitude of the physical quantity remains the same, whatever be the system of its measurement.

i.e. $$n_1u_1 = n_2u_2 \Rightarrow n_2 = \frac{n_1u_1}{u_2} \quad \ldots\text{(i)}$$

Where, $u_1$ and $u_2$ are two units of measurement of the quantity and $n_1$ and $n_2$ are their respective numerical values. If $M_1, L_1$ and $T_1$ are the fundamental units of mass, length and time in one system and while for other system, $M_2, L_2$ and $T_2$ are the fundamental units of mass, length and time then $u_1 = [M_1^a L_1^b T_1^c]$ and $u_2 = [M_2^a L_2^b T_2^c]$

From Eq. (i) $$n_2 = \frac{n_1[M_1^a L_1^b T_1^c]}{[M_2^a L_2^b T_2^c]} = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$\boxed{n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c}$$

**EXAMPLE |5| Energy Estimation**

A calorie is a unit of heat or energy and it equals about 4.2 J, where $1\text{ J} = 1\text{kg-m}^2/\text{s}^2$. Suppose we employ a system of units in which the unit of mass equals $\alpha$ kg, the unit of length is $\beta$ m, the unit of time is $\gamma$ s. Show that a calorie has a magnitude $4.2\,\alpha^{-1}\beta^{-2}\gamma^2$ in terms of new units.

[NCERT]

**Sol.** The dimensional formula of energy = $[ML^2T^{-2}]$

Let $M_1, L_1, T_1$ and $M_2, L_2, T_2$ are the units of mass, length and time in given two systems.

$\therefore$ $M_1 = 1\text{ kg}, M_2 = \alpha\text{ kg}$

$L_1 = 1\text{ m}, L_2 = \beta\text{ m}$

$T_1 = 1\text{ s}, T_2 = \gamma\text{ s}$

For any physical quantity, the product of its magnitude and unit is always constant.

$$n_1u_1 = n_2u_2$$

or $$n_2 = n_1\frac{u_1}{u_2} = 4.2 \times \frac{[M_1L_1^2T_1^{-2}]}{[M_2L_2^2T_2^{-2}]}$$

$$= 4.2\left[\frac{M_1}{M_2}\right]\times\left[\frac{L_1}{L_2}\right]^2\times\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 4.2\left[\frac{1}{\alpha}\text{kg}\right]\times\left[\frac{1}{\beta}\text{m}\right]^2\times\left[\frac{1}{\gamma}\text{s}\right]^{-2}$$

$$n_2 = 4.2\alpha^{-1}\beta^{-2}\gamma^2 \text{ new unit}$$

$\therefore$ $1\text{ cal} = 4.2\alpha^{-1}\beta^{-2}\gamma^2$ new unit

**EXAMPLE |6| Power Estimation**

Find the value of 60J per min on a system that has 100 g, 100 cm and 1 min as the base units.

**Sol.** Given, $P = \frac{60\text{ joule}}{1\text{min}} = \frac{60\text{ joule}}{60\text{ s}} = 1\text{ watt}$

which is the SI unit of power

Dimensional formula of power is $[ML^2T^{-3}]$.

$\therefore$ $a = 1, b = 2$ and $c = -3$

| SI | New System |
|---|---|
| $n_1 = 1$ | $n_2 = ?$ |
| $M_1 = 1\text{ kg} = 1000\text{ g}$ | $M_2 = 100\text{ g}$ |
| $L_1 = 1\text{ m} = 100\text{ cm}$ | $L_2 = 100\text{ cm}$ |
| $T_1 = 1\text{ s}$ | $T_2 = 1\text{ min} = 60\text{ s}$ |

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 1\left[\frac{1000}{100}\right]^{1}\left[\frac{100}{100}\right]^{2}\left[\frac{1}{60}\right]^{-3}$$

$$= 2.16\times10^6$$

$\therefore$ $60\text{ J min}^{-1} = 2.16\times10^6$ new units of power.

## 3. Deducing Relation among the Physical Quantities

The method of dimensions is used to deduce the relation among the physical quantities. We should know the dependence of the physical quantity on other quantities. We will explain with the following illustration.

Consider a simple pendulum having a bob attached to a string that oscillates under the action of the force of gravity.

Suppose, the time period $t$ of oscillation of the simple pendulum depends on its length ($l$), mass of the bob ($m$) and acceleration due to gravity ($g$).

Let $$t = km^a l^b g^c \quad \ldots\text{(i)}$$

Where a, $b$, $c$ are the dimensions and $k$ is dimensionless constant of proportionality.

Considering dimensions on both sides in terms of M, L, T, we get

$$[M^0L^0T^1] = M^aL^b[LT^{-2}]^c$$
$$= M^aL^{b+c}T^{-2c}$$

Applying the principle of homogeneity of dimensions, we get

$$a = 0, \; -2c = 1 \Rightarrow c = \frac{-1}{2},$$

$$b + c = 0 \Rightarrow b = -c \Rightarrow b = \frac{1}{2}$$

Substituting the values of $a$, $b$ and $c$ in Eq. (i), we get

$$t = km^0l^{1/2}g^{-1/2} = k\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad t = k\sqrt{\frac{l}{g}}$$

So, dimensional analysis is very useful in deducing relations among the interdependent physical quantities. It can only test the dimensional validity but not the exact relationship between physical quantities in any given equation.

### Problem Solving Strategy

(Derive an Expression)

1. Read the problem carefully and understand the concept of the problem before proceeding further.
2. Write all physical quantities which are known and unknown and list them.
3. Identify the physical parameter for all physical quantities.
4. Equation, the relationships between the physical quantities, should be written down next. Naturally, the selected equation should be consistent with the physical principles identified in the previous step.
5. Solve the set of equation for the unknown quantities in terms of the known. Do this algebraically, without substituting values until the next step ,except where terms are zero.
6. Substitute the known values, together with their units obtain a numerical value with units for each unknown.
7. Check your answer. Do the units match? Is the answer reasonable? Is your answer consistent with an order of magnitude estimate?

## Limitations of Dimensional Analysis

(i) It does not give any information whether a physical quantity is a scalar or a vector.

(ii) It gives no information about the dimensionless constant in the formula e.g. 1, 2, 3 … π etc.

(iii) We cannot drive the formula containing the trigonometrical function logarithmic function, exponential function which have no dimensions.

(iv) If a quantity depends on more than three factors, having dimensions, the formula cannot be derived.

This is because, equating the powers of M, L and T on either side of the dimensional equation, then we can obtain three equations from which we can compute three unknown dimensions.

**EXAMPLE |7| An Oscillating Bob**

Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on

(i) mass $m$ of the bob,

(ii) length $l$ of the pendulum and

(iii) acceleration due to gravity $g$ at the place.

Derive the expression for its time period using method of dimensions. **[NCERT]**

**Sol.** Let us assume that $T \propto m^a l^b g^c$

or $$T = km^a l^b g^c \qquad ...(i)$$

where, $k$ is a dimensionless constant.

The dimensions of various quantities are

$$[T] = \mathrm{T}, [m] = \mathrm{M}, [l] = \mathrm{L}, [g] = \mathrm{LT^{-2}}$$

Substituting these dimensions in Eq. (i), we get

$$\mathrm{T} = [\mathrm{M}]^a [\mathrm{L}]^b [\mathrm{LT^{-2}}]^c$$

or $$\mathrm{M^0 L^0 T^1} = \mathrm{M}^a \mathrm{L}^{b+c} \mathrm{T}^{-2c}$$

Equating the exponents of M, L and T on both sides, we get

$$a = 0, b + c = 0, -2c = 1$$

On solving, $a = 0, \quad b = \frac{1}{2}, \quad c = -\frac{1}{2}$

$\therefore$ $$T = km^0 l^{1/2} g^{-1/2} = k\sqrt{\frac{l}{g}}$$

From experiments, $k = 2\pi$

Therefore, $$T = 2\pi\sqrt{\frac{l}{g}}.$$

**EXAMPLE |8| A Stretched Spring**

A body of mass $m$ hung at one end of the spring executes SHM. Prove that the relation $T = 2\pi m/k$ is incorrect, where $k$ is the force constant of the spring. Also, derive the correct relation.

**Sol.** It is given that $T = \frac{2\pi m}{k}$

LHS, $\mathrm{T} = [\mathrm{T}]$

RHS, $$\frac{2\pi m}{k} = \frac{[\mathrm{M}]}{[\mathrm{MT^{-2}}]} = [\mathrm{T^2}]$$

$$\left[\because k = \frac{\text{Force}}{\text{Length}} = \frac{[\mathrm{MLT^{-2}}]}{[\mathrm{L}]} = [\mathrm{MT^{-2}}]\right]$$

$\therefore$ LHS ≠ RHS

Hence, the relation is incorrect.

To find the correct relation, suppose $T = km^a k^b$, then

$$[\mathrm{T}]^1 = [\mathrm{M}]^a [\mathrm{MT^{-2}}]^b = \mathrm{M}^{a+b} \mathrm{T}^{-2b}$$

$\therefore$ $$a + b = 0, -2b = 1$$

On solving, we get $b = \frac{-1}{2}, a = \frac{1}{2}$

$\therefore$ $$T = km^{1/2} k^{-1/2}$$

Hence, $$\mathrm{T} = k\sqrt{\frac{m}{k}}$$

**EXAMPLE |9| Vibration of Stretched String**

The frequency '$v$' of vibration of stretched string depends upon

(i) its length $l$,

(ii) its mass per unit length '$m$' and

(iii) the tension $T$ in the string

Obtain dimensionally an expression for frequency $v$.

**Sol.** Let the frequency of vibration of the string be given by

$$v = Kl^a m^b T^c \quad \text{...(i)}$$

where $K$ = a dimensionless constant

Dimensions of the various quantities are

$$v = [T^{-1}], l = [L], T = [T],$$

$$\text{Force} = [MLT^{-2}]$$

and $$m = \frac{\text{mass}}{\text{length}} = [ML^{-1}]$$

Substituting these dimensions in Eq. (i), we get

$$[T^{-1}] = [L]^a [ML^{-1}]^b [MLT^{-2}]^c$$

or $$[M^0L^0T^{-1}] = [M^{b+c}L^{a-b+c}T^{-2c}]$$

Equating the dimensions of M, L and T,

we get

$$b + c = 0, a - b + c = 0 \text{ and } -2c = -1$$

on solving, $$a = -1, b = -\frac{1}{2} \text{ and } c = \frac{1}{2}$$

$\therefore$ $$(v) = Kl^{-1}m^{-1/2}T^{1/2}$$

or $$(v) = \frac{K}{l}\sqrt{\frac{T}{m}}$$

**EXAMPLE |10| Time Period of Oscillation a Small Drop**

The time of oscillation $T$ of a small drop of a liquid under surface tension (whose dimensions are those of force per unit length) depends upon the density $d$, the radius $r$ and the surface tension $\sigma$. Derive dimensionally the relationship $T \propto \sqrt{dr^3/\sigma}$.

**Sol.** Let the time of oscillation of a small drop of a liquid is given by

$$T \propto d^a r^b d^c$$

$$T = kd^2r^2d^2 \quad \text{...(i)}$$

where, $K$ = a dimensionless constant

Dimension of the various quantities are

$$\sigma = [MT^{-2}], d = [ML^{-3}], r = [L]$$

Substituting these dimensions in Eq. (i), we get,

i.e. $$[M^0L^0T^1] = k[ML^{-3}]^a[L]^b[MT^{-2}]^c$$
$$= [M^{a+c}L^{-3a+b}T^{-2c}]$$

Equating the dimensions of M, L and T, we get,

$$a + c = 0, -2c = 1, -3a + b = 0$$

We get,

$$c = -1/2, a = 1/2 \text{ and } b = 3/2$$

$$T \propto d^{1/2}r^{3/2}\sigma^{-1/2}$$

i.e. $$T \propto \sqrt{\frac{dr^3}{\sigma}}$$

# TOPIC PRACTICE 3

## OBJECTIVE Type Questions

**1.** Which of the following has unit but no dimension?

(a) Angle (b) Strain

(c) Relative velocity (d) Relative density

**Sol.** (a) Angle has unit of radian but has no dimensions because, $\theta = \frac{l}{r}$

i.e., it is the ratio of two quantities of same dimensions.

**2.** Which of the following has same dimension as that of Planck constant?

(a) Work

(b) Linear momentum

(c) Angular momentum

(d) Impulse

**Sol.** (c) As, $E = h\nu$

or $$h = \frac{E}{\nu} = \left[\frac{ML^2T^{-2}}{T^{-1}}\right] = [ML^2T^{-1}]$$

Angular momentum $= mvr = [M][LT^{-1}][L] = [ML^2T^{-1}]$

**3.** Which of the following sets have different dimensions?

(a) Dipole moment, Electric field and Electric flux

(b) Pressure, Young's modulus, Stress

(c) Heat, Work, Energy

(d) Emf, Potential difference and potential

**Sol.** (a) Heat, work and energy are same things, so they have same dimensions.

Emf, potential difference and potential have the same dimensions.

$$\text{Pressure} = \frac{\text{force}}{\text{area}}, \text{stress} = \frac{\text{force}}{\text{area}}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{force / area}}{\text{dimensionless}} = \text{force / area}$$

So, they have same dimensions.

But dimension of Dipole moment $= [M^0L^1T^1A^1]$

dimenslion of electric field $= [M^1L^1T^{-3}A^{-1}]$

and dimension of electric flux $= [M^1L^3T^{-3}A^{-1}]$

hence they are different.

**4.** Obtain the dimensional equation for universal gas constant.

(a) $[ML^2T^{-2} mol^{-1}K^{-1}]$

(b) $[ML^3 T^{-1} mol^{-2} K^{-2}]$

(c) $[M^2LT^{-1} mol^{-1} K^{-1}]$

(d) $[M^3LT^{-2} mol^{-1} K^{-2}]$

**Sol.** (a) According to ideal gas equation for universal gas constant.

*i.e.*, $pV = nRT$, where $n$ is the number of moles of gases.

$$R = \frac{(p)(V)}{(n)(T)} = \frac{[ML^{-1} T^{-2}] [L^3]}{[mol] [K]}$$

$$= [ML^2T^{-2} mol^{-1}K^{-1}]$$

**5.** Given, force $= \dfrac{\alpha}{\text{Density} + \beta^3}$.

What are the dimensions of $\alpha$, $\beta$?

(a) $[ML^2 T^{-2}]$, $[ML^{-1/3}]$

(b) $[M^2L^4T^{-2}]$, $[M^{1/3}L^{-1}]$

(c) $[M^2L^{-2} T^{-2}]$, $[M^{1/3} L^{-1}]$

(d) $[M^2 L^{-2}T^{-2}]$, $[ML^{-2}]$

**Sol.** (c) Dimensions of $\beta^3$ = Dimensions of density = $[ML^{-3}]$

$$\beta = [M^{1/3} L^{-1}]$$

Also, $\alpha$ = Force $\times$ Density $= [MLT^{-2}] [ML^{-3}]$
$= [M^2L^{-2}T^{-2}]$

**6.** In the formula $x = 3yz^2$, $x$ and $z$ have dimensions of capacitance and magnetic induction, respectively. The dimensions of $y$ in MKS system are

(a) $[M^{-2} L^{-2} T^4A^4]$ (b) $[M^{-3} L^{-3} T^4A^5]$

(c) $[M^{-3} L^{-2} T^8A^4]$ (d) $[M^{-1}L^{-4}T^2A^4]$

**Sol.** (c) Given, $[x]$ = capacitance $= [M^{-1} L^{-2} T^4 A^2]$

$[z]$ = magnetic induction $= [MA^{-1} T^{-2}]$

So, $$[y] = \frac{[M^{-1}L^{-2}T^4A^2]}{[MA^{-1} T^{-2}]^2} = [M^{-3} L^{-2} T^8A^4]$$

**7.** When 1 m, 1 kg and 1 min are taken as the fundamental units, the magnitude of the force is 36 units. What will be the value of this force in CGS system?

(a) $10^5$ dyne (b) $10^3$ dyne (c) $10^8$ dyne (d) $10^4$ dyne

**Sol.** (b) As, dimensional formula of force $= [MLT^{-2}]$

$n_1 = 36$, $M_1 = 1$ kg, $L_1 = 1$ m, $T_1 = 1$ min $= 60$ s

$n_2 = ?$, $M_2 = 1$ g, $L_2 = 1$ cm, $T_2 = 1$ s

So, conversion of 36 units into CGS system

*i.e.*, $$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$n_2 = n_1 \left[\frac{1\text{ kg}}{1\text{g}}\right]^1 \left[\frac{1\text{ m}}{1\text{ cm}}\right]^1 \left[\frac{1\text{ min}}{1\text{ s}}\right]^{-2}$$

$$= 36\left[\frac{1000\text{ g}}{1\text{ g}}\right]\left[\frac{100\text{ cm}}{1\text{ cm}}\right]^1 \left[\frac{60\text{ s}}{1\text{ s}}\right]^{-2} = 10^3 \text{ dyne}$$

## VERY SHORT ANSWER Type Questions

**8.** Write the dimensional formula of wavelength and frequency of a wave.

**Sol.** Wavelength $[\lambda] = [L]$, Frequency $[\nu] = [T^{-1}]$

**9.** Obtain the dimensional formula for coefficient of viscosity.

**Sol.** Coefficient of viscosity $$(\eta) = \frac{Fdx}{A.dv} = \frac{[MLT^{-2}] [L]}{[L^2] [LT^{-1}]}$$

$$= [ML^{-1}T^{-1}]$$

**10.** What is the dimensional formula for torque?

**Sol.** $[ML^{-2}T^{-2}]$

**11.** Write three pairs of physical quantities, which have same dimensional formula.

**Sol.** (i) Work and energy (ii) Energy and torque
(iii) Pressure and stress

**12.** Express a joule in terms of fundamental unit.

**Sol.** Energy $= [ML^{-2}T^{-2}]$

Hence, $1\text{ J} = 1\text{ kg} \times 1\text{ m}^2 \times 1\text{ s}^{-2} = 1\text{kgm}^2\text{s}^{-2}$

**13.** Are all constants dimensionless?

**Sol.** No, it is not possible.

**14.** Name some physical quantities which are dimensionless.

**Sol.** Solid angle, relative density, strain, Reynold's number and Poisson's ratio.

**15.** Is Avogadro's number a dimensionless quantity?

**Sol.** No, it has dimensions. In fact, its dimensional formula is $[mol^{-1}]$.

**16.** If $x = a + bt + ct^2$, where $x$ is in metres and $t$ is second, what is the dimensional formula of $c$?

**Sol.** Here, $x = [L]$

$t = [T]$; $x = ct^2$

$[L] = c \times [T^2]$

$$\Rightarrow \frac{[L]}{[T^2]} = c \Rightarrow c = [LT^{-2}]$$

**17.** What are the dimensions of $a$ and $b$ in the relation : $F = a + bx$, where $F$ is force and $(x)$ is distance?

**Sol.** $$[a] = [F] = [MLT^{-2}], \quad [b] = \left[\frac{F}{x}\right] = \left[\frac{MLT^{-2}}{L}\right] = [MT^{-2}]$$

## SHORT ANSWER Type Questions

**18.** Find the dimensional formulae of (i) Kinetic energy and (ii) Pressure.

**Sol.** (i) $KE = \frac{1}{2}mv^2$ i.e., dimensional formula of KE is $[ML^2T^{-2}]$

(ii) Pressure $= \frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

**19.** State dimensional formulae for stress, strain and Young's modulus.

**Sol.** Strain is dimensionless quantity.

Dimensional formula for stress is $[ML^{-1}T^{-2}]$

Young's modulus has same dimensional formula as stress i.e. $[ML^{-1}T^{-2}]$

**20.** Using the relation $E = h\nu$, obtain the dimensions of Planck's constant.

**Sol.** We know that dimensional formula of energy $E$ of photon is $[M^1L^2T^{-2}]$ and dimensional formula of frequency $\nu$ is $[T^{-1}]$

$$[h] = \frac{[E]}{[\nu]} = \frac{[M^1L^2T^{-2}]}{[T^{-1}]} = [M^1L^2T^{-1}]$$

**21.** Magnitude of force $F$ experienced by a certain object moving with speed $v$ is given by $F = kv^2$, where $k$ is constant. Find the dimensions of $K$.

**Sol.** Since, $F = kv^2$.

Hence, $[k] = \frac{[F]}{[v^2]} = \frac{[MLT^{-2}]}{[LT^{-1}]^2} = [M^1L^{-1}]$

**22.** The rotational kinetic energy of a body is given by $E = \frac{1}{2}I\omega^2$, where $\omega$ is the angular velocity of the body. Use the equation to obtain dimensional formula for moment of inertia $I$. Also write its SI unit.

**Sol.** The given relation is $E = \frac{1}{2}I\omega^2$

$$I = \frac{[E]}{[\omega^2]} = \frac{[ML^2T^{-2}]}{[T^{-1}]^2}\left[\frac{ML^2T^{-2}}{T^{-2}}\right] = [ML^2]$$

Its SI unit is joule.

**23.** Distinguish between dimensional variables and dimensional constants. Give example too.

**Sol.** Dimensional variables are those quantities which have dimensions and whose numerical value may change. Speed, velocity, acceleration etc., are dimensional variables.

Dimensional constants are quantities having dimensions but having a constant value, e.g., gravitation constant ($G$), Planck's constant ($H$), Stefan's constant ($\sigma$) etc.

**24.** A book with many printing errors contains four different formulae for the displacement $y$ of a particle undergoing a certain periodic motion **[NCERT]**

(i) $y = a\sin\frac{2\pi t}{T}$ (ii) $y = a\sin vt$

(iii) $y = \left(\frac{a}{T}\right)\sin\left(\frac{t}{a}\right)$

(iv) $y = \left(\frac{a}{\sqrt{2}}\right)\left(\sin\frac{2\pi t}{T} + \cos\frac{2\pi t}{T}\right)$

(where, $a$ = maximum displacement of the particle, $v$ = speed of the particle, $T$ = time period of motion). Rule out the wrong formulae on dimensional grounds.

**Sol.** According to the principle of homogeneity of dimensions, if the dimensions of each term adding or subtracting in a given relation are same then it is correct, if not then it is wrong. The dimension of LHS of each relation is [L], therefore, the dimension of RHS should be [L] and the argument of the trigonometrical function i.e. angle should be dimensionless.

(i) As $\frac{2\pi t}{T}$ is dimensionless, therefore, dimension of RHS = [L]. This formula is correct.

(ii) Dimension of RHS = $[L]\sin[LT^{-1}][T] = [L]\sin[L]$

As angle is not dimensionless here. Therefore, this formula is wrong.

(iii) Dimension of RHS $= \frac{[L]}{[T]}\sin\frac{[T]}{[L]} = [LT^{-1}]\sin[TL^{-1}]$

As angle is not dimensionless here, therefore this formula is wrong.

(iv) Dimension of RHS $= [L]\left[\sin\frac{[T]}{[T]} + \cos\frac{[T]}{[T]}\right] = [L]$

As angle is dimensionless and dimension of RHS is equal to the dimension of LHS, therefore, this formula is correct.

**25.** A famous relation in Physics relates 'moving mass' m to the 'rest mass' $m_0$ of a particle in terms of its speed $v$ and speed of light $c$. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant $c$. He writes $m = \frac{m_0}{(1-v^2)^{1/2}}$. Guess, where to put the missing $c$? **[NCERT]**

**Sol.** The relation is written by the boy $m = \frac{m_0}{(1-v^2)^{1/2}}$

According to the principle of homogeneity of dimensions, the dimensions on either side of a relation must be same i.e., the powers of M, L, T on either side of a relation must be same.

Dimension of $m$ is equal to the dimension of $m_0$, therefore, the denominator $(1-v^2)^{1/2}$ should be dimensionless. In denominator 1 is dimensionless but factor $v^2$ is not dimensionless.

To make it dimensionless, we have to divide it by the same physical quantity with same power, therefore, it should be $v^2/c^2$, to become dimensionless.

Hence, the correct relation should be $m = \dfrac{m_0}{\left(1-\dfrac{v^2}{c^2}\right)^{1/2}}$

**26.** If $x = a + bt + ct^2$, where $x$ is in metre and $t$ in second, what are the units of $a, b$ and $c$?

***Sol.*** As $x = a + bt + ct^2$, where $x$ is in metre and $t$ in second. Hence, in accordance with the principle of homogeneity of dimensions, we have

Unit of $a = x =$ metre

Unit of $b =$ unit of $\dfrac{x}{t} =$ m/s and

Unit of $c =$ unit of $\dfrac{x}{t^2} = \text{m/(s)}^2$

**27.** A man walking briskly in rain with speed $v$ must slant his umbrella forward making an angle $\theta$ with the vertical. A student derives the following relation between $\theta$ and $v$: $\tan\theta = v$ and checks that the relation has a correct limit as $v \to 0$, $\theta \to 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation. [NCERT]

***Ans.*** Relation derived by the man, $\tan\theta = v$

The left hand side of the relation is dimensionless as it is a trigonometrical function.

The dimension of right hand of the relation = $[\text{LT}^{-1}]$

The dimension of LHS is not equal to the dimension of RHS. Therefore, this relation is not correct.

To be dimensionless, the RHS should be $\dfrac{v}{u}$.

Hence, the correct relation becomes $\tan\theta = \dfrac{v}{u}$

**28.** Check the correctness of the relation $v^2 - u^2 = 2as$ by method of dimensions. The symbol have their usual meaning.

***Sol.*** The relation is given as $v^2 - u^2 = 2as$

On LHS Dimension of $v^2 = [\text{L}^2\text{T}^{-2}]$

and $u^2 = [\text{LT}^{-1}]^2 = [\text{L}^2\text{T}^{-2}]$

RHS $2as = [\text{LT}^{-2}][\text{L}] = [\text{L}^2\text{T}^{-2}]$

As dimensions of both terms on LHS are equal to the dimensions of RHS, the relation is dimensionally correct.

**29.** The speed of sound in a solid is given by the formula

$$v = \sqrt{\frac{E}{\rho}}$$

where, $E$ is coefficient of elasticity and $\rho$ is density of given solid. Check the relation by method of dimensional analysis.

***Sol.*** In the given relation dimensions of LHS terms $v$ are $[\text{LT}^{-1}]$.

Dimensional formula for $E$ and $\rho$ are $[\text{ML}^{-1}\text{T}^{-2}]$ and $[\text{ML}^{-3}]$.

Dimensions of RHS $= \sqrt{\dfrac{\text{ML}^{-1}\text{T}^{-2}}{\text{ML}^{-3}}} = \sqrt{\text{L}^2\text{T}^{-2}} = [\text{LT}^{-1}]$

As dimensions of LHS and RHS of the equation are same. Hence the equation is dimensionally correct.

**30.** If force $F$, length $L$ and time $T$ are taken as fundamental units then what be the dimensions of mass?

***Sol.*** Suppose dimensions of mass $M$ be $[\text{F}^a\text{L}^b\text{T}^c]$. Then, we have

$$[\text{M}] = [\text{MLT}^{-2}]^a[\text{L}]^b[\text{T}]^c$$
$$= \text{M}^a\text{L}^{a+b}\,\text{T}^{-2a+c}$$
$$a = 1, a + b = 0, -2a + c = 0$$
$$b = -a = -1, c = 2a = 2$$

Hence, dimensions of mass $M$ are $[\text{F}^1\text{L}^{-1}\text{T}^2]$.

## LONG ANSWER Type I Questions

**31.** In the relation $p = (a/b)e^{-(aZ/\theta)}$, $p$ is the pressure, $Z$ is the distance, and $\theta$ is the temperature. What is the dimensional formula of $p$?

***Sol.*** Since, $e^{-(aZ/\theta)}$ is dimensionless, we have $aZ/\theta = 1$

or $$a = \frac{\theta}{Z} = \frac{K}{L} = [\text{L}^{-1}\text{K}]$$

We find that $a/b =$ dimensions of $p$ and $b = [\text{ML}^{-1}\text{T}^{-2}]$.

Therefore, dimensional formula of $p$ is obtained as

$$p = \frac{a}{[\text{ML}^{-1}\text{T}^{-2}]} = \frac{[\text{L}^{-1}\text{K}]}{[\text{ML}^{-1}\text{T}^{-2}]} = [\text{M}^{-1}\text{L}^0\text{T}^2\text{K}]$$

**32.** The SI unit of energy is $\text{J} = \text{kgm}^2\text{s}^{-2}$, that of speed $v$ is $\text{ms}^{-1}$ and acceleration $a$ is $\text{ms}^{-2}$. Which of the formulae for kinetic energy ($K$) given below can you rule out on the basis of dimensional arguments ($m$ stands for the mass of the body)?

(i) $K = m^2v^3$ (ii) $K = (1/2)mv^2$

(iii) $K = ma$ (iv) $K = (3/16)mv^2$

(v) $K = \left(\dfrac{1}{2}\right)mv^2 + ma$ [NCERT]

**Sol.** As SI unit of energy, $J = kgm^2s^{-2}$,

So, $[\text{energy}] = [ML^2T^{-2}]$

(i) $[m^2v^2] = [M^2][LT^{-1}]^2 = [M^2L^2T^{-2}]$

(ii) $[1/2m^2v^2] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

(iii) $[ma] = [M][LT^{-2}] = [MLT^{-2}]$

(iv) $[3/16mv^2] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

(v) The quantities $(1/2)mv^2$ and $ma$ have different dimensions and hence, cannot be added.

Since, the kinetic energy $K$ has the dimensions of $[ML^2T^{-2}]$, formulae (i), (iii) and (v) are clearly ruled out.

Dimensional analysis cannot tell which of the two, (ii) or (iv), is the correct formula. From the actual definition of kinetic energy, only (ii) is the correct formula for kinetic energy.

**33.** How will you convert a physical quantity from one unit system to another by method of dimensions?

**Ans.** If a given quantity is measured in two different unit system, then $Q = n_1u_1 = n_2u_2$

Let the dimensional formula of the quantity be $[M^aL^bT^c]$, then we have $n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$

Here $M_1, L_1, T_1$ are the fundamental unit of mass, length and time in first unit system and $M_2, L_2, T_2$ in the second unit system.

Hence, $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

This relation helps us to convert a physical quantity from one unit system to another.

**34.** Find the value of 60 W on a system having 100 g, 20 cm and 1 min as the fundamental units.

**Sol.** $n_1 = 60$ W, power is $[M^1L^2T^{-3}]$

In first system, $M_1 = 1$ kg, $L_1 = 1$ m, and $T_1 = 1$s

In second system, $M_2 = 100$ g, $L_2 = 20$ m,

and $T_2 = 1 \text{ min} = 60$ s

$$\text{So, } n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-3}$$

$$= 60\left[\frac{1000\text{ g}}{100\text{ g}}\right]\left[\frac{100\text{ cm}}{20\text{ cm}}\right]\left[\frac{1\text{ s}}{60\text{ s}}\right]^{-3}$$

$$= 60 \times \frac{1000}{100} \times \frac{100}{20} \times \frac{100}{20} \times 60 \times 60 \times 60$$

$$= 3.24 \times 10^9 \text{ units}$$

**35.** The wavelength $\lambda$ associated with a moving particle depends upon its mass $m$, its velocity $v$ and Planck's constant $h$. Show dimensional relation between them.

**Sol.** Suppose wavelength $\lambda$ associated with a moving particle depends upon (i) its mass ($m$), (ii) its velocity ($v$) and (iii) Planck's constant ($r$)

$$\lambda = km^av^bh^c$$

where, $k$ is a dimensionless constant.

Writing dimensions of various terms, we get

$$[M^0L^1T^0] = [M]^a[LT^{-1}]^b[ML^2T^{-1}]^c$$
$$= M^{a+c}L^{b+2c}T^{-b-c}$$

Comparing power of M, L and T on two sides of equation, we have

$$a + c = 0, b + 2c = 1, -b - c = 0$$

We get $a = -1, b = -1, c = +1$

Hence, the relation becomes $\lambda = \frac{kh}{mv}$

**36.** The orbital velocity $v$ of a satellite may depend on its mass $m$, distance $r$ from the centre of earth and acceleration due to gravity $g$. Obtain an expression for orbital velocity.

**Sol.** Suppose orbital velocity of satellite be given by the relation $v = km^ar^bg^c$

where, $k$ is a dimensionless constant and $a, b, c$ are unknown powers.

Writing dimensions on two sides of equation, we have

$$[M^0L^1T^{-1}] = [M]^a[L]^b[LT^{-2}]^c$$
$$= [M^aL^{b+c}T^{-2c}]$$

Applying principle of homogeneity of dimensional equation, we find that

$$a = 0 \Rightarrow b + c = 1 \Rightarrow -2c = -1$$

On solving these equations, we find that

$$a = 0, b = +\frac{1}{2} \text{ and } c = +\frac{1}{2}$$

$$v = kr^{1/2}g^{1/2} \Rightarrow v = k\sqrt{rg}$$

## LONG ANSWER Type II Questions

**37.** A large fluid star oscillates in shape under the influence of its own gravitational field. Using dimensional analysis, find the expression for period of oscillation ($T$) in terms of radius of star ($R$). Mean density of fluid ($\rho$) and universal gravitational constant ($G$).

**Sol.** Suppose period of oscillation $T$ depends on radius of star $R$, mean density of fluid $\rho$ and universal gravitational constant ($G$) as

$T = kR^a\rho^bG^c$, where $k$ is a dimensionless constant

$$[M^0L^0T^1] = [L]^a[ML^{-3}]^b[M^{-1}L^3T^{-2}]^c$$
$$= [M^{b-c}L^{a-3b+3c}T^{-2c}]$$

Comparing powers of M, L and T, we have

$$b - c = 0$$
$$a - 3b + 3c = 0 \quad \text{and} \quad -2c = 1$$

On simplifying these equations, we get

$$c = -1/2, \; b = -1/2, \; a = 0$$

Thus, we have $T = k\rho^{-1/2}G^{-1/2} = \frac{k}{\sqrt{\rho G}}$

**38.** Find an expression for viscous force $F$ acting on a tiny steel ball of radius $r$ moving in a viscous liquid of viscosity $\eta$ with a constant speed $v$ by the method of dimensional analysis.

***Sol.*** It is given that viscous force $F$ depends on (i) radius $r$ of steel ball, (ii) coefficient of viscosity $\eta$ of viscous liquid (iii) Speed $v$ of the ball

i.e., $F = kr^a \eta^b v^c$, where $k$ is dimensionless constant

Dimensional formula of force

$$F = [\text{MLT}^{-2}], \; r = [\text{L}]$$

$\eta = [\text{M}^1\text{L}^{-1}\text{T}^{-1}]$ and $v = [\text{LT}^{-1}]$, we have

$$[\text{MLT}^{-2}] = [\text{L}]^a [\text{M}^1\text{L}^{-1}\text{T}^{-1}]^b [\text{LT}^{-1}]^c$$
$$= [\text{M}^a \text{L}^{a-b+c} \text{T}^{-b-c}]$$

Comparing powers of M, L and T on either side of equation, we get

$$a = 1$$
$$a - b + c = 1$$
$$-b - c = -2$$

On solving, these above equations, we get

$$a = 1, \; b = 1 \text{ and } c = 1$$

Hence, the relation becomes

$$F = kr\eta v$$

**39.** A great physicist of this century (PAM Dirac) loved playing with the numerical values of fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics $m_e$, $m_p$ and the gravitational constant $G$, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (~15 billion yr).

From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants?

***Sol.*** Few basic constants of atomic physics are given below.

Charge of an electron $(e) = 1.6 \times 10^{-19}$C

Speed of light in vacuum $(c) = 3 \times 10^8$m/s

Gravitational constant $(G) = 6.67 \times 10^{-11}$ N-m$^2$/kg$^2$

Mass of electron $(m_e) = 9.1 \times 10^{-31}$ kg

Mass of proton $(m_p) = 1.67 \times 10^{-27}$ kg

Permittivity of free space $(\varepsilon_0) = 8.85 \times 10^{-12}$ N-m$^2$/C$^2$

On trying with these basic constants, we can get a quantity whose dimension is equal to the dimension of time. One such quantity is

$$x = \frac{e^4}{16\pi^2 \varepsilon_0^2 m_p m_e^2 c^3 G}$$

On writing dimensions of each quantity on RHS,

$$[x] = \frac{[\text{AT}]^4}{[\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]^2 \times [\text{M}] \times [\text{M}]^2 \times [\text{LT}^{-1}]^3 \times [\text{M}^{-1}\text{L}^3\text{T}^{-2}]}$$
$$= [\text{M}^{2-1-2+1}\text{L}^{6-3-3}\text{T}^{4-8+2+3}\text{A}^{4-4}]$$
$$= [\text{M}^{3-3}\text{L}^{6-6}\text{T}^{9-8}\text{A}^{4-4}]$$
$$= [\text{M}^0\text{L}^0\text{TA}^0] = [\text{T}]$$

Now, substituting values of all constants in the given relation,

$$x = \frac{(1.6 \times 10^{-19})^4}{16 \times (3.14)^2 \times (8.854 \times 10^{-12})^2 \times (1.67 \times 10^{-27}) \times (9.1 \times 10^{-31})^2 \times (3 \times 10^8)^3 \times (6.67 \times 10^{-11})}$$
$$= 2.18 \times 10^{16} \text{ s}$$
$$= 6.9 \times 10^8 \text{ yr} = 10^9 \text{ yr}$$
$$= 1 \text{ billion yr}$$

The estimates value of the quantity $x$ is close to the age of the universe.

# ASSESS YOUR TOPICAL UNDERSTANDING

## OBJECTIVE Type Questions

**1.** The dimensional formula of Avagadro's number is
(a) $[M^1L^1T^1]$
(b) $[mole^{-1}]$
(c) $[mole]$
(d) $[M^0L^1T^0]$

**2.** Dimension formula of $\Delta Q$, heat supplied to the system is
(a) $[ML^2T^{-2}]$ (b) $[MLT^{-2}]$
(c) $[ML^2T^{-1}]$ (d) $[MLT^1]$

**3.** Which of the following is not a dimensional constant?
(a) Gravitational constant
(b) $\pi$
(c) Planck's constant
(d) Gas constant ($R$)

**4.** Which of the following has neither units nor dimensions?
(a) Angle
(b) Energy
(c) Relative density
(d) Relative velocity

**5.** Which physical quantities have same dimension?
(a) Force and power
(b) Torque and energy
(c) Torque and power
(d) Force and torque

**6.** On checking the dimensional consistency of equation, it is based on the principle of
(a) homogeneity of equations
(b) homogeneity of dimensions
(c) homogeneity of expressions
(d) homogeneity of formula

**7.** Force ($F$) and density ($d$) are related as $F = \dfrac{\alpha}{\beta + \sqrt{d}}$.

Then, the dimensions of $\alpha$ and $\beta$ are
(a) $[M^{3/2}L^{-1/2}T^{-2}]$, $[ML^{-3}T^0]$
(b) $[M^{3/2}L^{-1/2}T^{-2}]$, $[M^{1/2}L^{-3/2}T^0]$
(c) $[M^2L^2T^{-1}]$, $[ML^{-1}T^{-3/2}]$
(d) $[MLT^{-2}]$, $[ML^{-2}T^{-2/3}]$

**8.** The density of a material in CGS system is 10 g cm$^{-3}$. If unit of length becomes 10 cm and unit of mass becomes 100 g, the new value of density will be
(a) 10 units (b) 100 units (c) 1000 units (d) 1 unit

### Answer

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (a) | 3. (b) | 4. (c) | 5. (b) |
| 6. (b) | 7. (b) | 8. (b) | | |

## VERY SHORT ANSWER Type Questions

**9.** Write the dimensional formula corresponding to (i) Photon and (ii) Calories.

**10.** Name at least seven physical quantities whose dimensions are $[ML^{-2}T^{-2}]$.

**11.** Write the dimensional formula of torque.

**12.** Name the physical quantity of the dimensions given that
(i) $[ML^{-1}T^{-1}]$, (ii) $[M^{-1}L^3T^{-2}]$,
(iii) $[ML^2T^{-3}]$ and (iv) $[ML^0T^{-2}]$

## SHORT ANSWER Type Questions

**13.** Write the dimensional formula of
(i) Thrust
(ii) Velocity gradient
(iii) Angular velocity

**14.** Write the dimensions of ($a$) Radius of gyration ($b$) Moment of force, ($c$) Moment of inertia, ($d$) Work done, ($e$) Strain, ($f$) Stress.

**15.** What are the advantages of expressing physical quantities in terms of dimensional equations?

**16.** How can you check the correctness of a dimensional equation?

**17.** Are all dimensionally correct equations numerically correct? Give one example.

**18.** Let us consider the equation $\frac{1}{2}mv^2 = mgh$, where $M$ is mass, $v$ is the velocity of the body, $g$ is the acceleration due to gravity and $h$ is the height. Check whether the equation dimensionally correct.
[**Ans.** Correct]

**19.** In CGS system, the value of Stefan's constant ($\sigma$) is $5.67 \times 10^{-5}$ ergs$^{-1}$cm$^{-2}$K$^{-4}$. Write down its value in SI units. [**Ans.** $5.67 \times 10^{-8}$ Js$^{-1}$m$^{-2}$K$^{-4}$]

## LONG ANSWER Type I Questions

**20.** Find the dimensions of constant '*a*' and '*b*' occurring in van der Waals' equation.

$$\left[p + \frac{a}{V^2}\right][V - b] = RT$$

[**Ans.** $a = [M^1 L^5 T^{-2}], b = [L^3]$]

**21.** In Poiseuille's equation, $V = \pi p r^4 / 8\eta l$, determine the dimension of $\eta$.

**22.** Write the dimensions of $a$ and $b$ in the relation $E = b - x^2/at$, where $E, x$ and $t$ represent energy, distance and time, respectively.

**23.** The resistivity $\rho$ of the material depends on the length $l$, diameter $d$ and resistance $R$ of the wire. Derive the relation for resistivity using the method of dimensions.

$\left[\textbf{Ans. } \rho = k\left(\frac{Rd^2}{l}\right)\right]$

## LONG ANSWER Type II Questions

**24.** Assume that the mass ($M$) of the largest stone that car moved by the following river depends only upon the velocity $v$ and the density $\rho$ of the water, alongwith the acceleration due to gravity $g$. Show that $m$ varies with sixth power of the velocity of the flow. [**Ans.** $M = kv^6$]

**25.** The escape velocity $v$ of a body depends upon (i) the acceleration due to gravity of the planet and (ii) the radius of the planet $R$. Establish dimensionally the relationship $v$, $g$ and $R$. [**Ans.** $v = k\sqrt{gR}$]

**26.** Explain the principle of homogeneity of dimensions. What are its uses? Illustrate by giving one example of each.

**27.** Using the principle of homogeneity of dimensions find which of the following is correct.

(i) $T^2 = 4\pi^2 r^2$, (ii) $T^2 = \frac{4\pi^2 r^3}{G}$ and

(iii) $T^2 = \frac{4\pi^2 r^3}{4M}$

# SUMMARY

- All the quantities in terms of which laws of physics are described and whose measurement is necessary, are called physical quantities.
- **Types of physical quantities**
  - (i) **Fundamental quantities** The physical quantities which can be treated as independent of other physical quantities and are not usually defined in terms of other physical quantities are called fundamental quantities. e.g. mass, length , time etc.
  - (ii) **Derived quantities** Those physical quantities which are derived from fundamental quantities are called derived quantities. e.g. velocity, acceleration, force etc.
- **Unit** The standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a physical unit.
- **Types of physical units**
  - (i) **Fundamental units** The physical units which can neither be derived from one another, nor they can be further resolved into more simpler units are called fundamental units. e.g. kg, metre, second etc.
  - (ii) **Derived units** All other units which can be expressed in terms of fundamental units are called derived units e.g. m/s, $m/s^2$, etc.
- The comparision of any physical quantity with its standard unit is called **measurment**.
- **Rules for finding significant figures**
- The dimensions of a physical quantity are powers (or exponents) to which the base quantities are raised to represent the quantity.
- The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the dimensional formula.
- The equation which expresses a physical quantity in terms of the fundamental units of mass, length and time is called dimensional equation.
- If the dimensions of left hand side of an equation are equal to the dimensions of right hand side of the equation, then the equation is dimensionally correct. This is known as homogeneity principle. Mathematically [LHS] = [RHS]

# CHAPTER PRACTICE

## OBJECTIVE Type Questions

**1.** Which one of the following physical quantities is not a fundamental quantity?
(a) Luminous intensity
(b) Thermodynamic temperature
(c) Electric current
(d) Work

**2.** Among the given following system of unit which is not based on unit of mass, length and time?
(a) CGS (b) FPS (c) MKS (d) SI

**3.** Find the value of 12.9 g – 7.05 g.
(a) 5.84 g (b) 5.8 g
(c) 5.86 g (d) 5.9 g

**4.** The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give **[NCERT Exemplar]**
(a) 2.75 and 2.74 (b) 2.74 and 2.73
(c) 2.75 and 2.73 (d) 2.74 and 2.74

**5.** The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is
(a) $164 \pm 3\ cm^2$ (b) $163.62 \pm 2.6\ cm^2$
(c) $163.6 \pm 2.6\ cm^2$ (d) $163.62 \pm 3\ cm^2$

**6.** If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s. What is the unit of mass in this system of units?
(a) $10^5$ kg (b) $10^7$ kg
(c) $10^2$ kg (d) $10^9$ kg

**7.** If $P, Q, R$ are physical quantities, having different dimensions, which of following combinations can never be meaningful quantity?
(a) $\left(\frac{P-Q}{R}\right)$ (b) $PQ - R$
(c) $\frac{PQ}{R}$ (d) $\frac{PR - Q^2}{R}$

**8.** If $R$ and $L$ represent resistance and self-inductance respectively, which of the following combinations has the dimensions of frequency?
(a) $\frac{R}{L}$ (b) $\frac{L}{R}$
(c) $\sqrt{\frac{R}{L}}$ (d) $\sqrt{\frac{L}{R}}$

**9.** In the gas equation $\left(p + \frac{a}{V^2}\right)(V - b) = RT$, the dimensions of $a$ are
(a) $[ML^3T^{-2}]$ (b) $[M^{-1}L^3T^{-1}]$
(c) $[ML^5T^{-2}]$ (d) $[M^{-1}L^{-5}T^2]$

## ASSERTION AND REASON

**Direction** (Q. Nos. 10-14) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

**10.** **Assertion** The unit based for measuring nuclear cross-section is 'barn'.
**Reason** 1 barn $= 10^{-14} m^2$.

**11.** **Assertion** Pressure has the dimensions of energy density.
**Reason** Energy density $= \frac{\text{Energy}}{\text{Volume}}$

**12.** **Assertion** The method of dimensions analysis cannot validate the exact relationship between physical quantities in any equation.
**Reason** It does not distinguish between the physical quantities having same dimensions.

**13.** **Assertion** Let us consider an equation

$$(1/2)\, mv^2 = mgh$$

where, $m$ is the mass of the body, $v$ is velocity, $g$ is the acceleration due to gravity and $h$ is the height.
**Reason** Equation is dimensionally correct.

**14.** **Assertion** A cesium atomic clock is used at NPL, New Delhi to maintain the Indian standard of time.

**Reason** The cesium atomic clocks are very accurate and precise.

## CASE BASED QUESTIONS

**Direction** (Q. Nos. 15-16) *This question is case study based question. Attempt any 4 sub-parts from given question.*

**15.** **Measuring Experiment**

The true value of a certain length is near 3.678 cm. In one experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 3.5 cm, while in another experiment using a measuring device of greater resolution, say 0.01 cm, the length is determined to be 3.38 cm.

(i) Which measuring experiment is more accurate?
(a) First (b) Second
(c) Either (a) or (b) (d) Neither (a) nor (b)

(ii) Which measuring experiment is more precise?
(a) First (b) Second
(c) Either (a) or (b) (d) Neither (a) nor (b)

(iii) A device which is used for measurement of length to an accuracy of about $10^{-4}$m is
(a) screw gauge
(b) spherometer
(c) vernier calliper
(d) Either (a) or (b)

(iv) Instrument contains some uncertainty. This uncertainty is
(a) error
(b) mistake
(c) precision
(d) accuracy

(v) The accuracy in measurement may depend on
(a) limit, resolution
(b) precision, limit
(c) limit, accuracy
(d) precision, accuracy

**16.** **Dimensions Analysis**

All quantities in mechanics are represented in terms of base units of length, mass and time. Additional base unit of temperature (kelvin) is used in heat and thermodynamics. In magnetism and electricity, the additional base unit of electric current is ampere.

(i) The dimensions of universal gravitational constant are
(a) $[ML^{-3}T^{2}]$ (b) $[ML^{2}T^{-3}]$
(c) $[M^{-1}L^{3}T^{-2}]$ (d) $[M^{2}L^{2}T^{-2}]$

(ii) The coefficient of thermal conductivity has the dimensions
(a) $[ML^{-1}T^{3}K^{3}]$ (b) $[ML^{-1}T^{-3}K^{-1}]$
(c) $[MLT^{-3}K^{-1}]$ (d) $[MLT^{-3}K]$

(iii) Dimensions of resistance are
(a) $[MLT^{-3}A^{-1}]$ (b) $[ML^{2}T^{-3}A^{-1}]$
(c) $[M^{2}LT^{-3}A^{-1}]$ (d) $[ML^{2}T^{-3}A]$

(iv) Given, force $= \frac{\alpha}{\text{Density} + \beta^3}$.
What are the dimensions of $\alpha, \beta$?
(a) $[ML^{2}T^{-2}], [ML^{-1/3}]$
(b) $[M^{2}L^{4}T^{-2}], [M^{1/3}L^{-1}]$
(c) $[M^{2}L^{-2}T^{-2}], [M^{1/3}L^{-1}]$
(d) $[M^{2}L^{-2}T^{-2}], [ML^{-2}]$

(v) Which of the following has unit but no dimension?
(a) Angle (b) Strain
(c) Relative velocity (d) Relative density

### Answer

| | | | | |
|---|---|---|---|---|
| 1. (d) | 2. (d) | 3. (b) | 4. (d) | 5. (a) |
| 6. (a) | 7. (a) | 8. (a) | 9. (c) | 10. (c) |
| 11. (a) | 12. (a) | 13. (a) | 14. (a) | |

15. (i) (a) (ii) (b) (iii) (d) (iv) (a) (v) (a)
16. (i) (c) (ii) (c) (iii) (b) (iv) (c) (v) (a)

## VERY SHORT ANSWER Type Questions

**17.** Wavelength of a laser light is 6463 Å. Express it in nm and micron. [**Ans.** 646.3 nm, 0.6463 μ]

**18.** Find the value of 1 J of energy in CGS system of units. [**Ans.** $10^7$ CGS units]

## SHORT ANSWER Type Questions

**19.** Find the value of the following upto the appropriate significant figures in the following.
(a) 3.27 + 33.5472 (b) 53.312 − 53.3
(c) $2.02 \times 23$ (d) $3.908 \times 5.5$
[**Ans.** (a) 36.82 (b) zero (c) 46 (d) 21]

20. If unit of force, velocity and energy are 100 dyne, 10 cm/s and 400 ergs, respectively. What will be the unit of mass, length and time?

[**Ans.** 4 g, 4 cm, 0.4]

21. If $p$ represents radiation pressure, $c$ represents the speed of light and $q$ represents the radiation energy per unit area per unit time.

Calculate the non-zero integers such that $p^x q^y c^z$ is dimensionless.

## LONG ANSWER Type I Questions

22. How many significant figures are there in the following results for quantities measured in the laboratory?

(a) 3.0120 (b) 123.0 km (c) 0.006235 s

(d) $0.23 \times 10^{-3}$ (e) 100.007 g (f) 143000 km

23. If velocity of sound in air $v$ depends on the modulus of elasticity $E$ and density $\rho$.

Find expression of $v$. $\left[\textbf{Ans. } v = k\sqrt{\frac{E}{\rho}}\right]$

24. Check the relation $S = ut + \frac{1}{2}at^2$ by method of dimension.

25. If the time period ($T$) of vibration of a liquid drop depends on surface tension ($s$) and radius ($r$) of the drop, and density ($\rho$) of the liquid. Derive an expression for $T$ using dimensional analysis. $\left[\textbf{Ans. } T = \frac{pr^3}{s}\right]$

26. The depth $x$ to which a bullet penetrates a human body depends upon (i) coefficient of elasticity $\eta$ and (ii) kinetic energy $E_k$. By the method of dimensions, show that $x \propto \left[\frac{E_k}{\eta}\right]^{1/3}$

27. Rule out or accept the following formula for kinetic energy on the basis of dimensional arguments.

(i) $R = \frac{3}{16}mv^2$ (ii) $K = \frac{1}{2}mv^2 + ma$

[**Ans.** $x = 1,\ y = -1,\ z = 1$]

28. Miss Rita in her Physics class was explaining how to use a vernier calliper to measure small dimension. Her student Anubha was confused that why didn't we used a metre scale for the purpose. She asked Miss Rita who explained her that a proper instrument and unit should be used to get accurate measurement. Large dimensions such as distance of earth from Sun is measured in light year (ly) whereas size of nucleus is measured in fermi (fm).

(i) What values of Anubha and her teacher do you appreciate?

(ii) Give the physical quantities of following units.

(a) Parsec (b) Joule

(iii) Give the physical quantities measured by following instrument

(a) Barometer (b) Pyrometer

## LONG ANSWER Type II Question

29. Specific resistance $\rho$ of a thin circular wire of radius $r$ in cm, resistance $R$ in $\Omega$ and length $L$ in cm is given by $\rho = (\pi r^2 R)/L$. If $r = (0.26 \pm 0.01)$ cm, $R = (30 \pm 2)\,\Omega$ and $L = (75.00 \pm 0.01)$ cm, find the percentage error in $\rho$ upto correct significant figures. [**Ans.** 14%]